Preliminary version:

Thm 6.2.1: Hint: use integration by parts and let $dv = f'(t)$

Thm 6.3.2: Hint: Let $F(s) = \mathcal{L}(f(t)) =$ .... To calculate $F(s-c)$ evaluate $F(s)$ at $s-c$ (i.e. replace s with $s-c$). Use definition of Laplace transform to evaluate

Thm: The Laplace transform is a linear operator.
Hint: $\mathcal{L}(af(t) + bg(t)) = ...$ OR [$\mathcal{L}(af(t)) = ...$ and $\mathcal{L}(f(t) + g(t)) = ...$]

Thm: $L(f) = af'' + bf' + cf$ is linear.
Hint: $L(k_1 f(t) + k_2 g(t)) = ...$ OR [$L(kf(t)) = ...$ and $L(f(t) + g(t)) = ...$]

Thm 3.2.2: If $y = \phi_1(t)$ and $y = \phi_2(t)$ are solutions to $y'' + p(t)y + q(t)y = 0$, then $y = c_1\phi_1(t) + c_2\phi_2(t)$ is also a solution to this homogeneous equation.

Thm 7.4.1: If $\overrightarrow{y} = \overrightarrow{\phi_1}(t)$ and $\overrightarrow{y} = \overrightarrow{\phi_2}(t)$ are solutions to $\overrightarrow{y}' = P(t)\overrightarrow{y}$, then $\overrightarrow{y} = c_1\overrightarrow{\phi_1}(t) + c_2\overrightarrow{\phi_2}(t)$ is also a solution to this homogeneous equation.

Thm 3.5.1: If $y = \psi_1(t)$ and $y = \psi_2(t)$ are solutions to $y'' + p(t)y + q(t)y = g(t)$, then $y = \psi_2(t) - \psi_1(t)$ is a solution to the homogeneous equation $y'' + p(t)y + q(t)y = 0$.

Note 3.5.1 plus previous thms imply $\psi_2(t) - \psi_1(t) = c_1\phi_1 + c_2\phi_2$ for some constants c_1, c_2 where ϕ_1, ϕ_2 are 2 linearly independent solutions to the homogeneous equation.

Hence $\psi_2(t) = c_1\phi_1 + c_2\phi_2 + \psi_1(t)$. I.e., if you can find the general homogeneous solution $(c_1\phi_1(t) + c_2\phi_2(t))$ and one non-homogeneous, then you can find all non-homogeneous solutions to $y'' + p(t)y + q(t)y = g(t)$. I.e., this non-homogeneous equation has general solution: $y = c_1\phi_1 + c_2\phi_2 + \psi_1(t)$.