22M:100 (MATH:3600:0001) Final Exam
May 17, 2013

SHOW ALL STEPS

[13] 1.) Solve: $ty' + 6t^2y - t^2 = 0$, $y(1) = 3$

Answer: ______________________________

[13] 2.) Solve: $(2x^3y^2 - 3x)dx + (x^4y - y^{-1})dy = 0$

Answer: ______________________________

[14] 3.) Use the infinite series solution method (ch 5) about the point $x_0 = 0$ to solve the differential equation, $(x+1)y' - 2y = 0$.

Note: You must use the infinite series method. However, this is still a relatively short problem as most terms will cancel out (so no need for induction proof/ratio test).

Answer: ________________________________

[20] 4.) Solve: $y'' - 3y' - 4y = 8t$, $y(0) = 5$, $y'(0) = 3$

Answer: ____________________________________

[20] 5.) Suppose $\frac{dx}{dt} = (x-3)(y+2)$ and $\frac{dy}{dt} = 2x - y$.

(a.) State all critical points of this system of differential equations ______________________

(b.) State two solutions to this system of differential equations.

(c.) For the critical point with NEGATIVE x coordinate, find the corresponding linear system.

(d.) Based on the linear translated approximation found in part c, what conclusions regarding stability can you draw about the nonlinear system near the critical point with negative x coordinate?

[20] 6.) Choose 2 from the following three problems (6A, 6B, 6C). Clearly indicate your TWO choices. You may do more than 2 problems in which case I may substitute your unchosen problem for one of your two choices (with a penalty) if it improves your grade.

Your two choices: ______________________________

6A.) Suppose $y = \phi_1(t)$, $y = \phi_2(t)$, and $y = \phi_3(t)$ are all solutions to the homogeneous differential equation $y''' + p(t)y'' + q(t)y' + r(t)y = 0$ and suppose that $y = \psi(t)$ is a solution to $y''' + p(t)y'' + q(t)y' + r(t)y = g(t)$. Prove that $y = c_1\phi_1(t) + c_2\phi_2(t) + c_3\phi_3(t) + \psi(t)$ is a solution to $y''' + p(t)y'' + q(t)y' + r(t)y = g(t)$.

6B.) Suppose $a_n = \frac{-2a_{n-1}}{n+1}$ for all $n \geq 1$. Use induction to prove that $a_n = \frac{(-2)^n a_0}{(n+1)!}$ for all $n \geq 0$

6C.) Transform the given initial value problem into an equivalent problem with initial point at the origin:

$$\frac{dy}{dt} = 2t - 3(y+1)^2,\ y(8) = 10$$

[5 Extra Credit] Prove the following theorem for up to 5 points extra credit.

Suppose M, N, M_y, N_x are continuous. Prove that $M(x,y) + N(x,y)y' = 0$ is an exact differential equation if and only if $M_y = N_x$. Furthermore prove that $M(x,y) + N(x,y)y' = 0$ has a solution if it is exact.

Note you may earn partial credit by proving part of the above or just for starting the proof, stating definitions, etc..