Lemma: Two unoriented rational knots $N(\frac{a_1}{b_1})$ and $N(\frac{a_2}{b_2})$, $a_i \geq 0$, are the same iff $a_1 = a_2$ and $b_1 b_2^{\pm 1} \cong 1 \pmod{a_1}$.

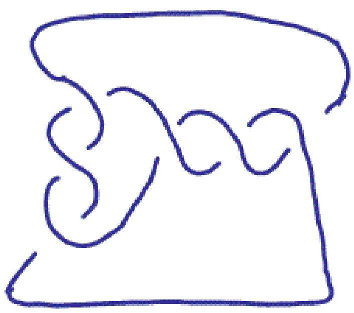
Ex: $N(\frac{10}{3}) = N(\frac{10}{7}) = N(\frac{10}{7+10k}) = N(\frac{10}{3}) = N(\frac{10}{3+10k})$

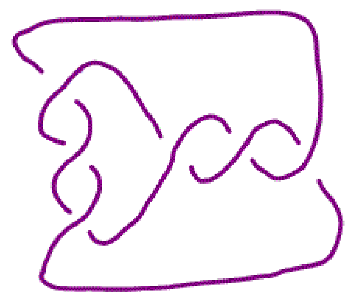
since $7(3) = 1 \bmod 10$.

Observe $N(\frac{10}{3})$ is an achiral knot since it is equivalent to its mirror image, i.e., $N(\frac{10}{3}) = N(\frac{10}{-3})$.

$\frac{10}{3} = 3 + \frac{1}{3}$

$\frac{10}{3} = -3 + \frac{1}{-3}$





In general, $$\left(x_n + \frac{1}{x_{n-1}+\cdots\frac{1}{x_1}}\right) = x_n + \frac{1}{x_{n-1}+\cdots\frac{1}{x_1}}$$

Thus $N(\frac{a}{-b})$ is the mirror image of $N(\frac{a}{b})$.

Lemma: Suppose $\begin{vmatrix} d & j \\ q & p \end{vmatrix} = pd - qj = 1$. Then

$$N\left(\frac{j}{p} + \frac{t}{w}\right) = N\left(\frac{jw + pt}{dw + qt}\right)$$

Example: Calculate $N(\frac{8}{5} + \frac{3}{2})$

Observe $\begin{vmatrix} -3 & 8 \\ -2 & 5 \end{vmatrix} = 1, \quad \begin{vmatrix} 5 & 8 \\ 3 & 5 \end{vmatrix} = 1, \quad \begin{vmatrix} -1 & 3 \\ -1 & 2 \end{vmatrix} = 1.$

$N(\frac{8}{5} + \frac{3}{2}) = N(\frac{16+15}{-3(2)-2(3)}) = N(\frac{31}{-12})$

$N(\frac{8}{5} + \frac{3}{2}) = N(\frac{31}{5(2)+3(3)}) = N(\frac{31}{19})$

$N(\frac{8}{5} + \frac{3}{2}) = N(\frac{31}{-1(8)-1(5)}) = N(\frac{31}{-13})$

Observe:

-12 = 19 mod 31 since 19 - (-12) = 31 = 0 mod 31

(-12)(-13) = 156 = 156 - 93 = 63 = 0 mod 31.

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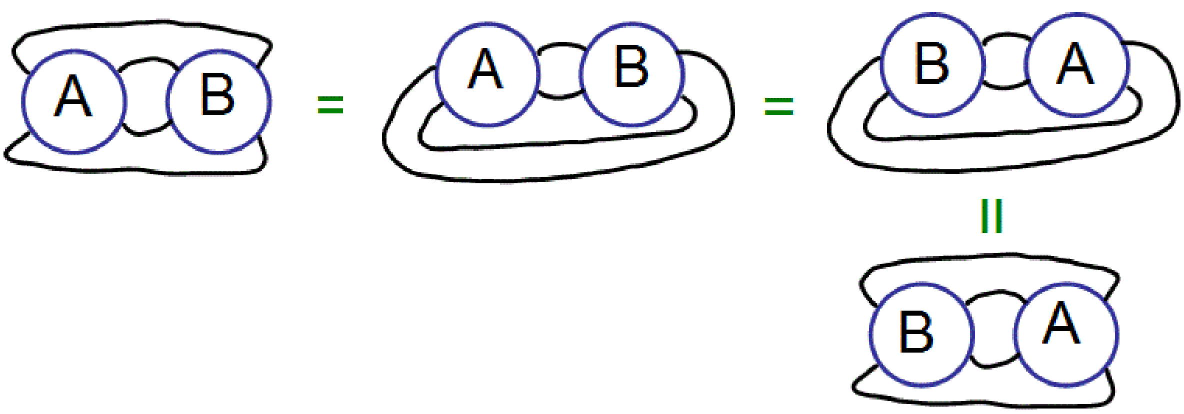
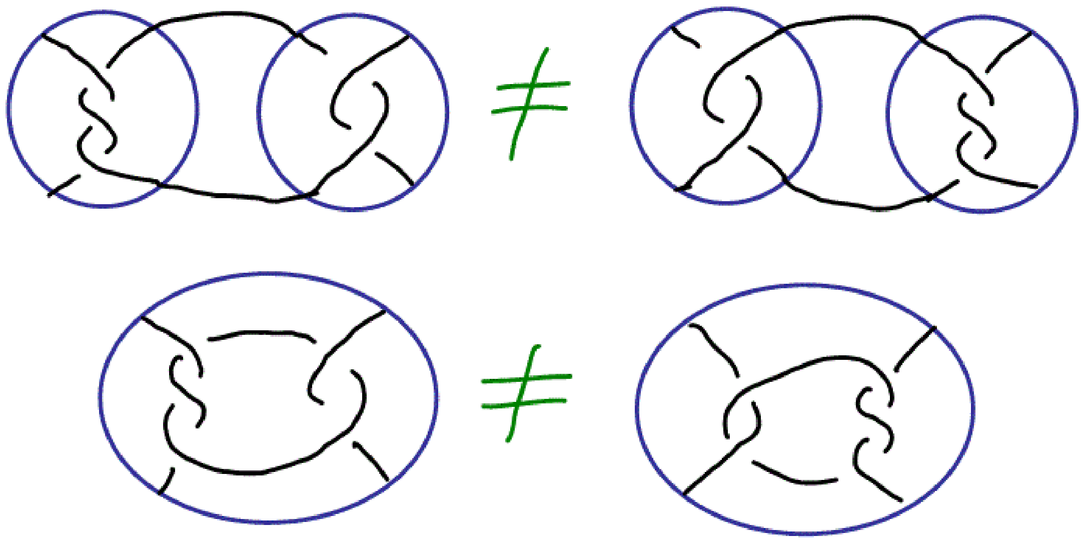
Lemma: $N(A + B) = N(B + A)$



Lemma: Tangle addition is not commutative:

$$A + B \neq B + A$$



$$\frac{1}{3} + \frac{1}{-2} \neq \frac{1}{-2} + \frac{1}{3}$$

Goal: Given $a, b, z, v$ where $b \in \{0, 1, ..., a-1\}$, solve the system of 2 equations:

$$N\left(\frac{j}{p} + \frac{0}{1}\right) = N\left(\frac{a}{b}\right) \qquad (*)$$

$$N\left(\frac{j}{p} + \frac{t}{w}\right) = N\left(\frac{z}{v}\right) \qquad (**)$$

for $\frac{j}{p}$ and $\frac{t}{w}$

Project: Given $a_i, b_i, z_i, v_i,\ \ i = 1, 2$, solve the system of 4 equations:

$$N\left(\frac{j_1}{p_1} + \frac{0}{1}\right) = N\left(\frac{a_1}{b_1}\right) \qquad (*)$$

$$N\left(\frac{j_1}{p_1} + \frac{t}{w}\right) = N\left(\frac{z_1}{v_1}\right) \qquad (**)$$

$$N\left(\frac{j_2}{p_2} + \frac{0}{1}\right) = N\left(\frac{a_2}{b_2}\right) \qquad (***)$$

$$N\left(\frac{j_2}{p_2} + \frac{t}{w}\right) = N\left(\frac{z_2}{v_2}\right) \qquad (****)$$

HW:

1.) $N(\frac{4}{9} + 3) =$

2.) $N(\frac{4}{9} + \quad 3) =$

3.) $N(\frac{4}{9} + \frac{5}{3}) =$

4.) $N(\frac{4}{9} + \frac{10}{3}) =$

5.) Solve $N\left(\frac{j}{p} + \frac{0}{1}\right) = N\left(\frac{1}{0}\right)$, $N\left(\frac{j}{p} + \frac{t}{w}\right) = N\left(\frac{4}{1}\right)$.

6.) Solve $N\left(\frac{j}{p} + \frac{0}{1}\right) = N\left(\frac{6}{1}\right)$, $N\left(\frac{j}{p} + \frac{t}{w}\right) = N\left(\frac{15}{4}\right)$.