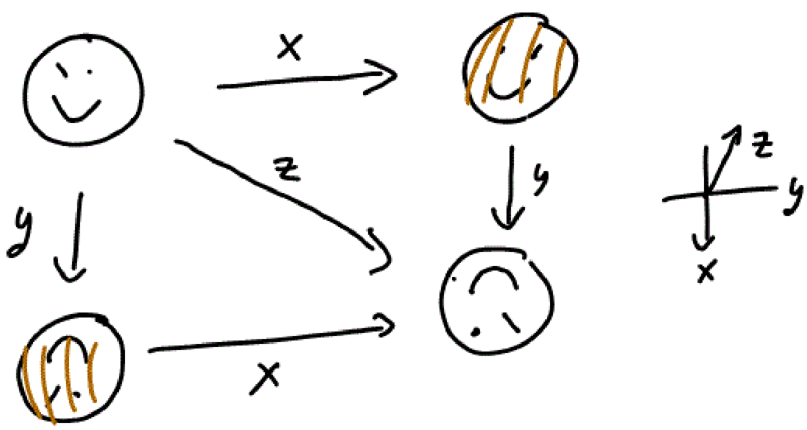
Symmetries of Rational tangles / links

Note Title 3/31/2010

Thm 1: If T = ☺ is a rational 2-string tangle, then T is invariant under 180° rotations about the x, y, z-axis. That is the following tangles are all equivalent:



Reference: ADVANCES IN APPLIED MATHEMATICS **18,** 300-332 1997. Rational Tangles by Jay Goldman, Louis H. Kauffman

Pf: Note invariance under 180° about x and about y-axis ⇒ invariance about z-axis by 180°

Let $T = (c_1, \dots, c_n)$

Proof by induction on n

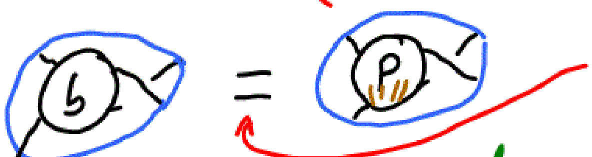
WLOG assume n is odd.

$n=1$: Note  are invariant under 180° rotation about x & y-axis.

Suppose (b) is invariant under a 180° rotation about x-axis & about y-axis.

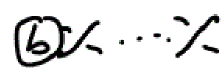
I.e. (b) = (d) = (p)

flipping (b)



only but this is equiv to flipping (b)

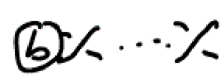
 is invariant

Thus by induction on k, under 180° about y-axis. Similarly for 

ambient isotopy

moving inside string

 is invariant

Thus by induction on k, 180° about ~~y~~-axis. Similarly for 

(⟶ chalkboard)

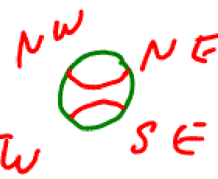
Let G = symmetry group of $N\left(\frac{a}{b}\right)$ where a is even

I.e.: the action of any element of G on $N(a/b)$ does not change its link type

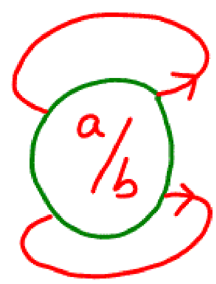
I.e $N\left(\frac{a}{b}\right) \xrightarrow[g]{} L$ then

$$N\left(\frac{a}{b}\right) = L \text{ iff } g \in G$$

Recall $N\left(\frac{a}{b}\right)$ is a link $\Longleftrightarrow$ a is even $\Longleftrightarrow$ $\frac{a}{b}$ has parity 0:

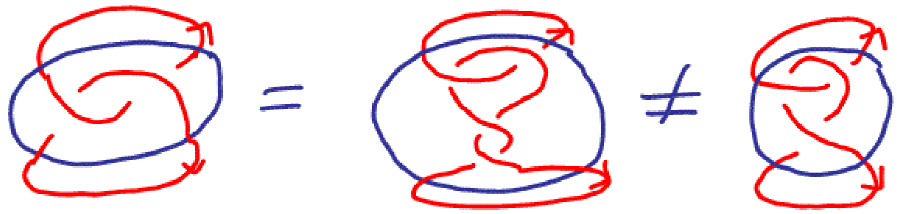


Defn:
The default orientation of $N\left(\frac{a}{b}\right)$ is



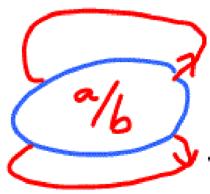
Thm $N(a/b) = N(a/d)$ as oriented links $\iff bd^{\pm 1} = 1 \mod 2a$

Ex



$N\left(\frac{2}{1}\right) = N\left(\frac{2}{5}\right) \neq N(2/3)$

Defn:



Thm: $N\left(\frac{a}{b}\right) \xrightarrow[(1,1,-1,e)]{} N\left(\frac{a}{b+a}\right)$

reverse the orientation of the 2nd component.

Thm: $N\left(\frac{a}{b}\right) \xrightarrow[(-1,1,1,e)]{} N\left(-\frac{a}{b}\right)$

take mirror image