6.5: Impulse functions

Unit impulse function = Dirac delta function is a generalized function with the properties

$$\delta(t) = 0, \qquad t \neq 0$$

$$\int_{-\infty}^{\infty} \delta(t)dt = 1$$

Let $d_\tau(t) = \begin{cases} \frac{1}{2\tau} & -\tau < t < \tau \\ 0 & t \leq -\tau \text{ or } t \geq \tau \end{cases}$

Note $lim_{\tau\to 0} d_\tau(t) = 0$ if $t \neq 0$

and $lim_{\tau\to 0} \int_{-\infty}^{\infty} d_\tau(t) = lim_{\tau\to 0} = 1 = \int_{-\infty}^{\infty} \delta(t)dt$

$$\mathcal{L}(\delta(t - t_0)) = lim_{\tau\to 0}\mathcal{L}(d_\tau(t - t_0))$$

$$= lim_{\tau\to 0} \int_0^{\infty} e^{-st} d_\tau(t - t_0)dt$$

$$= lim_{\tau\to 0} \frac{1}{2\tau} \int_{t_0-\tau}^{t_0+\tau} e^{-st}dt$$

$$= lim_{\tau\to 0} \frac{-1}{2s\tau} e^{-st}\Big|_{t_0-\tau}^{t_0+\tau}$$

$$= lim_{\tau\to 0} \frac{1}{2s\tau} e^{-st_0}(e^{s\tau} - e^{-s\tau})$$

$$= lim_{\tau\to 0} \frac{sinhs\tau}{s\tau} e^{-st_0} = lim_{\tau\to 0} \frac{scoshs\tau}{s} e^{-st_0}$$

$$= e^{-st_0}$$