The LaPlace Transform is a method to change a differential equation to a linear equation.

Example: Solve $y'' + 3y' + 4y = 0$, $y(0) = 5$, $y'(0) = 6$

1.) Take the LaPlace Transform of both sides of the equation:

$$\mathcal{L}(y'' + 3y' + 4y) = \mathcal{L}(0)$$

2.) Use the fact that the LaPlace Transform is linear:

$$\mathcal{L}(y'') + 3\mathcal{L}(y') + 4\mathcal{L}(y) = 0$$

3.) Use thm to change this equation into an algebraic equation:

$$s^2\mathcal{L}(y) - sy(0) - y'(0) + 3[s\mathcal{L}(y) - y(0)] + 4\mathcal{L}(y) = 0$$

3.5) Substitute in the initial values:

$$s^2\mathcal{L}(y) - 5s - 6 + 3[s\mathcal{L}(y) - 5] + 4\mathcal{L}(y) = 0$$

4.) Solve the algebraic equation for $\mathcal{L}(y)$

$$s^2\mathcal{L}(y) - 5s - 6 + 3s\mathcal{L}(y) - 15 + 4\mathcal{L}(y) = 0$$

$$[s^2 + 3s + 4]\mathcal{L}(y) = 5s + 21$$

$$\mathcal{L}(y) = \frac{5s+21}{s^2+3s+4}$$

Some algebra implies $\mathcal{L}(y) = \frac{5s+21}{s^2+3s+4}$

5.) Solve for y by taking the inverse LaPlace transform of both sides (use a table):

$$\mathcal{L}^{-1}(\mathcal{L}(y)) = \mathcal{L}^{-1}\left(\frac{5s+21}{s^2+3s+4}\right)$$

$$y = \mathcal{L}^{-1}\left(\frac{5s+21}{s^2+3s+4}\right)$$

Use partial fractions or completing the square to find $\mathcal{L}^{-1}\left(\frac{5s+21}{s^2+3s+4}\right)$:

$$b^2 - 4ac = 3^2 - 4(1)(4) < 0$$

$$\frac{5s+21}{s^2+3s+4} =$$