

Tuesday, March 30, 2010
Kauffman Bracket Polynomial

$$\langle \begin{array}{c} A \\ \diagdown \quad \diagup \\ B \quad B \\ \diagup \quad \diagdown \\ A \end{array} \rangle = A \langle \rangle \langle \rangle + B \langle \frown \rangle$$

$$\langle 0, k \rangle = d \langle k \rangle$$

↑ ↑ ↑
unlinked knot some
unknotted coefficient
component

Must also have the following required relation:

$$\langle \rangle \langle \rangle = \langle \bigcirc \rangle = A^2 \langle \bigcap \rangle + AB \langle \bigodot \rangle + BA \langle \bigcup \rangle + B^2 \langle \bigcup \rangle$$

state = (x, y) $\left. \begin{array}{l} x=0,1 \\ y=0,1 \end{array} \right\}$ vertices of a hypercube

$$= A^2(0,0) + AB(0,1) + BA(1,0) + B^2(1,1)$$

$$(1-BA) \langle \rangle \langle \rangle = (A^2 + ABd + B^2) \langle \frown \rangle$$

$$1-BA=0$$

$$A^2 + ABd + B^2 = 0$$

$$B = A^{-1}$$

$$d = -A^2 - A^{-2}$$

R3:

$$\langle \begin{array}{c} \diagdown \quad \diagup \\ \diagup \quad \diagdown \end{array} \rangle = A \langle \begin{array}{c} \diagdown \quad \diagup \\ \diagup \quad \diagdown \end{array} \rangle + B \langle \begin{array}{c} \diagdown \quad \diagup \\ \diagup \quad \diagdown \end{array} \rangle$$

$$\langle \begin{array}{c} \diagdown \quad \diagup \\ \diagup \quad \diagdown \end{array} \rangle = A \langle \begin{array}{c} \diagdown \quad \diagup \\ \diagup \quad \diagdown \end{array} \rangle + B \langle \begin{array}{c} \diagdown \quad \diagup \\ \diagup \quad \diagdown \end{array} \rangle$$

R2 \Rightarrow R3

Thus, $\langle \rangle$ is invariant under R2 & R3

It's also an invariant of framed links (R1 moves not allowed)

$$\begin{aligned}
 R1: \langle \begin{array}{c} B \\ A \diagdown \diagup \\ A \end{array} \rangle &= A \langle \begin{array}{c} \cup \\ \cup \end{array} \rangle + B \langle \begin{array}{c} \cup \\ \cap \end{array} \rangle \\
 &= A(-A^2 - A^{-2}) \langle \begin{array}{c} \cup \\ \cup \end{array} \rangle + A^{-1} \langle \sim \rangle \\
 &= -A^3 \langle \sim \rangle
 \end{aligned}$$

Thus, $\langle \rangle$ is not a knot invariant

Similarly, $\langle \begin{array}{c} \nearrow \\ \searrow \end{array} \rangle = -A^{-3} \langle \sim \rangle$

$X(L) = (-A^3)^{-\text{wr}(L)} \langle L \rangle$ is a knot invariant.

$$A \mapsto t^{-1/4}$$

$X(L) \mapsto$ Jones polynomial of L

$$\langle \begin{array}{c} \diagdown \\ \diagup \end{array} \rangle = A \langle \begin{array}{c} \diagdown \\ \diagdown \end{array} \rangle + A^{-1} \langle \begin{array}{c} \cup \\ \cap \end{array} \rangle$$

$$\langle \begin{array}{c} \diagup \\ \diagdown \end{array} \rangle = A^{-1} \langle \begin{array}{c} \cup \\ \cap \end{array} \rangle + A \langle \begin{array}{c} \diagup \\ \diagup \end{array} \rangle$$

$$\Rightarrow A \langle \begin{array}{c} \nearrow \\ \searrow \end{array} \rangle - A^{-1} \langle \begin{array}{c} \searrow \\ \nearrow \end{array} \rangle = (A^2 - A^{-2}) \langle \begin{array}{c} \cup \\ \cap \end{array} \rangle$$

$$X(L) = (-A^3)^{-\text{wr}(L)} \langle L \rangle$$

$$= (-A^3)^{-\text{wr}(L)} \sum \langle L_s \rangle$$

states \uparrow all crossings resolved

$\prod_{[0,1]^n}$

state sum

* Handout given *

See following example in handout:

$$\pi_1(S^3 \setminus \{z\}) = \langle x, y, z \mid z = x^{-1}yx, y = z^{-1}xz \rangle$$

$$\cong \langle x, y \mid y = z^{-1}xz \text{ where } z = x^{-1}yx \rangle$$

$$\cong \langle xyx^{-1} \mid yx^{-1} = z^{-1}xz^{-1} \rangle$$

Let $a = yx^{-1}$.

$$\langle x, a \mid a = \underbrace{x^2 a^{-1} x^{-1}}_{z^{-1}} (x^{-1} a x^2) x^{-1} \rangle$$

$$\cong \langle x, a \mid 1 = a^{-1} (x^{-2} a^{-1} x^2) (x^{-1} a x) \rangle$$

↓ abelianized

$$(\alpha \mid 0 = -\alpha - \alpha t^{-2} + \alpha t^{-1})$$

$$\Rightarrow \Lambda / (t^2 - t + 1)$$

Note: $a = yx^{-1} \leftarrow \alpha$

$$t^{-1}\alpha \rightarrow x^{-1}ax = x^{-1}yx^{-1}x = x^{-1}y$$

$$t^{-2}\alpha \rightarrow x^{-2}ax^2 = x^{-1}z$$

