Tuesday, March 30, 2010

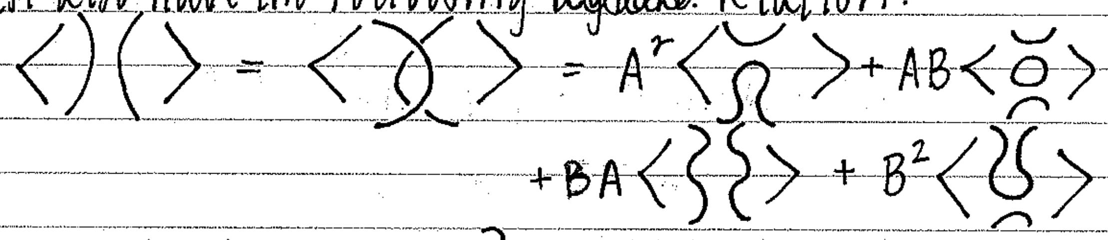
Kauffman Bracket Polynomial

$$\left\langle {}_{B}\overset{A}{\underset{A}{\times}}{}_{B} \right\rangle = A\left\langle)(\right\rangle + B\left\langle \asymp \right\rangle$$

$$\langle O, K \rangle = d\langle K \rangle$$

unlinked unknotted component — knot — some coefficient

Must also have the following ~~[illegible]~~ relation:



state = (x, y) $x = 0, 1$, $y = 0, 1$ } vertices of a hypercube

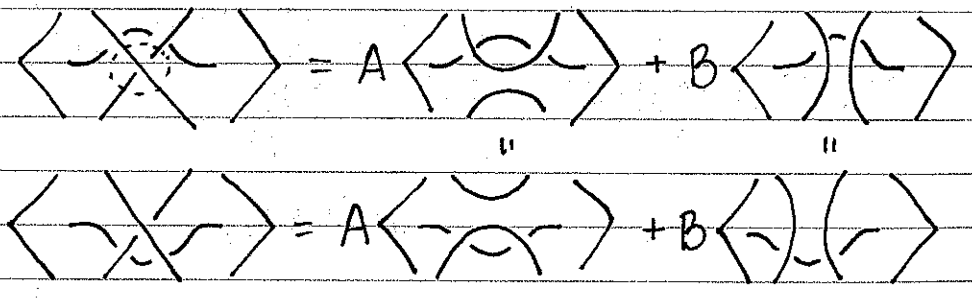
$$= A^2(0,0) + AB(0,1) + BA(1,0) + B^2(1,1)$$

$$(1-BA)\langle)(\rangle = (A^2 + ABd + B^2)\langle \asymp \rangle$$

$1 - BA = 0$ $A^2 + ABd + B^2 = 0$

$B = A^{-1}$ $d = -A^2 - A^{-2}$

R3 :

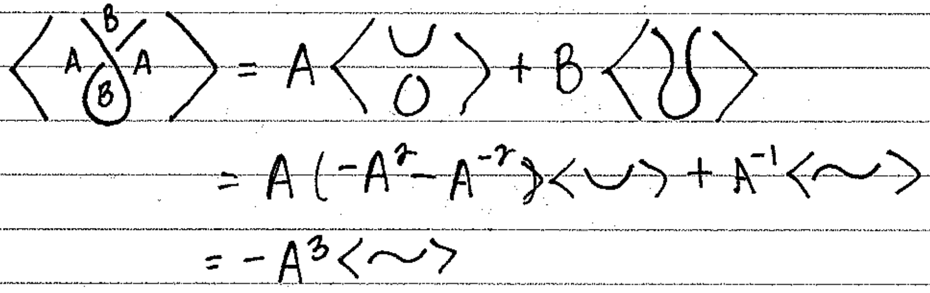


R2 ⇒ R3

Thus, $\langle\ \rangle$ is invariant under R2 & R3

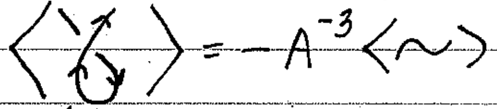
It's also an invariant of framed links (R1 moves not allowed)

R1:



Thus, $\langle\ \rangle$ is not a knot invariant

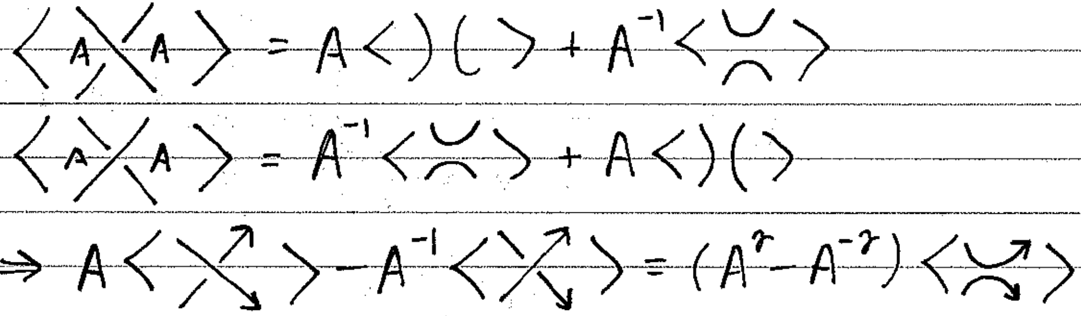
Similarly,



$X(L) = (-A^3)^{-wr(L)}\langle L\rangle$ is a knot invariant.

$A \mapsto t^{-1/4}$

$X(L) \mapsto$ Jones polynomial of L



$$X(L) = (-A^3)^{-wr(L)}\langle L\rangle$$
$$= \underbrace{(-A^3)^{-wr(L)} \sum_{\substack{\text{states}\\ \Pi[0,1]^n}} \langle L_s\rangle}_{\text{state sum}}$$

↖ all crossings resolved

Handout given

See following example in handout:

$$\pi_1(S^3 \setminus 3_1) = \langle x, y, z \mid z = x^{-1} y x,\ y = z^{-1} x z \rangle$$

$$\cong \langle x, y \mid y = z^{-1} x z \text{ where } z = x^{-1} y x \rangle$$

$$\cong \langle x y x^{-1} \mid y x^{-1} = z^{-1} x z x^{-1} \rangle$$

Let $a = y x^{-1}$.

$$\langle x, a \mid a = \underbrace{x^{2} a^{-1} x}_{z^{-1}}\, x (x^{-1} a x^{2}) x^{-1}$$

$$\cong \langle x, a \mid 1 = a^{-1} (x^{-2} a^{-1} x^{2}) (x^{-1} a x) \rangle$$

$\downarrow$ abelianized

$$(\alpha \mid 0 = -\alpha - \alpha t^{-2} + \alpha t^{-1})$$

$$\Rightarrow \Lambda / (t^{2} - t + 1)$$

Note: $a = y x^{-1} \leftarrow \alpha$

$t^{-1}\alpha \rightarrow x^{-1} a x = x^{-1} y x^{-1} x = x^{-1} y$

$t^{-2}\alpha \rightarrow x^{-2} a x^{2} = x^{-1} z$