Suppose the density of an object is given by $f(x, y, z) = x^2z+y+1$ where the object is described by $x^2 + y^2 \leq z \leq x - 2y + 3$

$x^2+y^2 \leq z \leq x-2y+3$ describes the region, W, below the plane $z = x - 2y + 3$ and above the paraboloid $z = x^2 + y^2$

Note mass = (density)(volume)

Thus mass $= \int\int\int_W f(x, y, z)dV$
$= \int_{c_1}^{c_2}[\int_{g_1(x)}^{g_2(x)}[\int_{f_1(x,y)}^{f_2(x,y)}(x^2z + y + 1)dz]dy]dx$

$x^2 + y^2 \leq z \leq x - 2y + 3$

The plane and the parabola intersect at the following domain values: $x^2 + y^2 = x - 2y + 3$.

Thus $y^2 + 2y + x^2 - x - 3 = 0$.
Thus for a fixed x,
y ranges between $y = \frac{-2\pm\sqrt{4-4(x^2-x-3)}}{2} = -1 \pm \sqrt{-x^2 + x + 4}$.

Hence $-1 - \sqrt{-x^2 + x + 4} \leq y \leq -1 + \sqrt{-x^2 + x + 4}$.

We need to find the minimum and maximum values of x when $x^2 + y^2 = x - 2y + 3$.

$x^2 - x + y^2 + 2y - 3 = 0$.

Thus $x = h(y) = \frac{1\pm\sqrt{1-4(y^2+2y-3)}}{2}$

$h'(y) = \pm\frac{1}{4}[1 - 4(y^2 + 2y - 3)]^{\frac{-1}{2}}(-8y - 8)$

$h'(y) = 0$ iff $y = -1$. In this case $x = h(-1) = \frac{1\pm\sqrt{1-4(1-2-3)}}{2} = \frac{1\pm\sqrt{1-4(-4)}}{2} = \frac{1\pm\sqrt{17}}{2}$

$h'(y)$ DNE iff $1 - 4(y^2 + 2y - 3) = 0$ (note $1 - 4(y^2 + 2y - 3 < 0)$ is not in the domain of h, so those y values are immaterial.

Thus potential max and min's for x are: $x = h(y) = \frac{1}{2}, \frac{1\pm\sqrt{17}}{2}$

Hence $\frac{1-\sqrt{17}}{2} \leq x \leq \frac{1+\sqrt{17}}{2}$

Alternatively,
note $y^2 + 2y + x^2 - x = 3$ implies $(x - \frac{1}{2})^2 + (y + 1)^2 = \frac{17}{4}$

Thus mass $= \int\int\int_W f(x, y, z)dV$
$= \int_{c_1}^{c_2}[\int_{g_1(x)}^{g_2(x)}[\int_{f_1(x,y)}^{f_2(x,y)}(x^2z + y + 1)dz]dy]dx$

$$= \int_{\frac{1-\sqrt{17}}{2}}^{\frac{1+\sqrt{17}}{2}}[\int_{1-\sqrt{1-x^2+x+3}}^{1+\sqrt{1-x^2+x+3}}[\int_{x^2+y^2}^{x-2y+3}(x^2z + y + 1)dz]dy]dx$$