Define: $f : X \to Y$ is 1:1 iff $\underline{f(x) = f(y) \text{ implies } x = y}$

Define: $f : X \to Y$ is NOT 1:1 iff
$\underline{\text{there exists } x \neq y \text{ such that } f(x) = f(y)}$

Prove that $f : R \to R$, $f(x) = x^4$ is NOT 1:1.

Define: $f : X \to Y$ is onto iff $\underline{f(X) = Y}$.

Define: $f : X \to Y$ is NOT onto iff $\underline{\text{there exists } y \in Y}$ $\underline{\text{s. t. there does not exist an } x \in X \text{ s. t. } f(x) = y}$ (i.e., y is not in the image of f).

Prove that $f : R \to R$, $f(x) = x^4$ is NOT onto.

State the Intermediate Value Theorem: Suppose f continuous on $[a, b]$, $f(a) \neq f(b)$ and N is between $f(a)$ and $f(b)$, then there exists $c \in (a, b)$ such that $f(c) = N$.

Use the Intermediate Value Theorem to show that there exists $c \in [1, 2]$ such that $c^4 = 2$.

Use the Intermediate Value Theorem to show that there exists $c \in [-2, -1]$ such that $c^4 = 2$.

Defn: $lim_{x \to a} f(x) = L$ if
for all $\epsilon > 0$, there exists a $\delta > 0$ such that
$0 < |x - a| < \delta$ implies $|f(x) - L| < \epsilon$

Prove $lim_{x \to 2} 3x = 6$

Take $\epsilon > 0$. Choose $\delta = \frac{\epsilon}{3}$

Suppose $0 < |x - 2| < \frac{\epsilon}{3}$

Claim $|3x - 6| < \epsilon$

Proof of claim: $|3x - 6| = 3|x - 2| < 3(\frac{\epsilon}{3}) = \epsilon$

Define f continuous at a:

Define f differentiable at a:

Prove that if f differentiable at a, then f is continuous at a.

Hint: Show $lim_{x \to a}[f(x) - f(a)] = 0$

Thm: If f is differentiable at x and c is a constant, then $(cf)'(x) = c(f'(x))$