Exam 1 March 3, 2011 — SHOW ALL WORK

22M:025 Calculus I — Either circle your answers or place on answer line.

[5] 1.) State the limit definition of the derivative: $f'(x) = \underline{lim_{h\to 0}\frac{f(x+h)-f(x)}{h}}$

[7] 2.) Choose one of the following (**clearly indicate your choice: 2A or 2B**).

2A.) Prove: If c is a constant and f is differentiable at x, then $(cf)'(x) = c(f'(x))$

$(cf)'(x) = lim_{h\to 0}\frac{cf(x+h)-cf(x)}{h} = lim_{h\to 0}\frac{c(f(x+h)-f(x))}{h} = c(lim_{h\to 0}\frac{f(x+h)-f(x)}{h}) = c(f'(x))$

2Bi.) Define: $f : X \to Y$ is 1:1 iff

$f(x) = f(y)$ implies $x = y$

ii.) Define: $f : X \to Y$ is NOT 1:1 iff

If there exists $x \neq y$ such that $f(x) = f(y)$

iii.) Prove that $f : R \to R$, $f(x) = \sqrt{x^2}$ is NOT 1:1.

$f(-2) = 2 = f(2)$

[10] 3.) Suppose $f(x) = e^x$. Evaluate the following (FILL in the blank) and graph:

a.) Graph $y = f(x + 2) = \underline{e^{x+2}}$

b.) Graph $y = f(-x) = \underline{e^{-x}}$

c.) Graph $y = 2f(x - 1) - 3 = \underline{2e^{x-1} - 3}$

d.) Graph $y = f^{-1}(x) = \underline{ln(x)}$

[10] 4.) Simplify and express the given quantity as a single logarithm:

$$\big[ln(e^2 - 1) - ln(e - 1)\big] \cdot [ln(e^2)ln(e) - ln(1)] = \underline{ln[(e + 1)^2]}$$

$\big[ln(e^2 - 1) - ln(e - 1)\big] \cdot [ln(e^2)ln(e) - ln(1)] = \big[ln\frac{(e^2-1)}{(e-1)}\big] \cdot [(2)(1) - 0] = \big[ln(e + 1)\big] \cdot (2) =$

$2\big[ln(e + 1)\big] = ln[(e + 1)^2]$

5.) Let $f(x) = \frac{\sqrt{3x^6-1}}{x^3-x^2-x+1} = \frac{\sqrt{3x^6-1}}{(x-1)^2(x+1)}$

[5] 5a.) The domain of f is $\underline{(-\infty,-1)\cup(-1,-(\frac{1}{3})^{\frac{1}{6}}]\cup[(\frac{1}{3})^{\frac{1}{6}},1)\cup(1,\infty)}$

$x \neq 1, -1$ and $3x^6 - 1 \geq 0$

Hence $x^6 \geq \frac{1}{3}$

Hence $[x \geq (\frac{1}{3})^{\frac{1}{6}}$ or $x \leq -(\frac{1}{3})^{\frac{1}{6}}]$ and $[x \neq 1, -1]$.

[5] 5b.) Show all steps: $lim_{x\to-\infty} f(x) = \underline{-\sqrt{3}}$

$lim_{x\to-\infty} \frac{\sqrt{3x^6-1}}{x^3-x^2-x+1} =$

$lim_{x\to-\infty} \frac{\sqrt{3x^6-1}}{x^3(1-\frac{1}{x}-\frac{1}{x^2}+\frac{1}{x^3})} =$

Since x is negative, $x^3 = -\sqrt{x^6}$

$\frac{\sqrt{3x^6-1}}{x^3} = \frac{\sqrt{3x^6-1}}{-\sqrt{x^6}} = -\sqrt{\frac{3x^6-1}{x^6}} = -\sqrt{3-\frac{1}{x^6}}$

$lim_{x\to-\infty} \frac{\sqrt{3x^6-1}}{x^3(1-\frac{1}{x}-\frac{1}{x^2}+\frac{1}{x^3})} = lim_{x\to-\infty} \frac{-\sqrt{3-\frac{1}{x^6}}}{(1-\frac{1}{x}-\frac{1}{x^2}+\frac{1}{x^3})} = -\sqrt{3}$

[5] 5c.) $lim_{x\to+\infty} f(x) = \underline{\sqrt{3}}$

[4] 5d.) Does $y = f(x)$ have any horizontal asymptotes? $\underline{yes}$. If so, state the equation(s) of all horizontal asymptote(s):

$y = -\sqrt{3}$ and $y = \sqrt{3}$

5cont.) Recall $f(x) = \frac{\sqrt{3x^6-1}}{x^3-x^2-x+1} = \frac{\sqrt{3x^6-1}}{(x-1)^2(x+1)}$

[5] 5e.) $lim_{x\to1} f(x) = \underline{+\infty}$

$\frac{+}{(+0)(+)}$

[5] 5f.) $lim_{x\to-1} f(x) = \underline{DNE}$

[5] 5g.) $lim_{x\to-1^-} f(x) = \underline{-\infty}$

$\frac{+}{(+)(-0)}$

[4] 5h.) Does $y = f(x)$ have any vertical asymptotes? $\underline{yes}$. If so, state the equation(s) of all vertical asymptote(s):

$x = -1$ and $x = 1$

[10] 6.) If $f(x) = \frac{x^2-1}{e^x sinx}$, then $f'(x) = \underline{\frac{2xe^x sinx-(x^2-1)[e^x sinx+e^x cosx]}{(e^x sinx)^2}}$

$f'(x) = \frac{2xe^x sinx-(x^2-1)[e^x sinx+e^x cosx]}{(e^x sinx)^2}$

[10] 7.) Find equation of tangent line to $f(x) = sin(4x-3)+2$ at $x = 1$

$f'(x) = 4cos(4x-3)$

slope $= f'(1) = 4cos(1)$

$f(1) = sin(1) + 2$

slope $= f'(1) = 4cos(1) = \frac{y-(sin(1)+2)}{x-1}$

$4cos(1)(x-1) = y - sin(1) - 2$

$4xcos(1) - 4cos(1) + sin(1) + 2 = y$

Answer: $\underline{y = 4xcos(1) - 4cos(1) + sin(1) + 2}$

[10] 8.) If $3xy = \sqrt{y} + x$, then $\frac{dy}{dx} = \underline{\frac{6y^{\frac{3}{2}}-2y^{\frac{1}{2}}}{1-6xy^{\frac{1}{2}}}}$

$3y + 3xy' = \frac{1}{2}y^{-\frac{1}{2}}y' + 1$

$3y - 1 = \frac{1}{2}y^{-\frac{1}{2}}y' - 3xy'$

$3y - 1 = [\frac{1}{2}y^{-\frac{1}{2}} - 3x]y'$

$y' = \frac{3y-1}{\frac{1}{2}y^{-\frac{1}{2}}-3x} = \frac{6y-2}{y^{-\frac{1}{2}}-6x} = \frac{6y^{\frac{3}{2}}-2y^{\frac{1}{2}}}{1-6xy^{\frac{1}{2}}}$