3.8

Find the derivative of $(3x+4)^{2x}$

Method 1: use implicit differentiation and logarithmic differentiation:

Write as an equation and "simplify" until the equation is in a format in which you know how to take the derivative:

$y = (3x+4)^{2x}$ Find y'

$ln(y) = ln(3x+4)^{2x}$

$ln(y) = 2x \cdot ln(3x+4)$

Now that we have an equation where we know how to take the derivative of both sides, we can take the derivative using implicit differentiation:

$\frac{1}{y}y' = 2ln(3x+4) + 2x(\frac{1}{3x+4})3$

$y' = y[2ln(3x+4) + \frac{6x}{3x+4}] = (3x+4)^{2x}[2ln(3x+4) + \frac{6x}{3x+4}]$

Method 2: Use logarithmic differentiation directly:

$(3x+4)^{2x} = e^{ln(3x+4)^{2x}}$

Thus $[(3x+4)^{2x}]' = [e^{ln(3x+4)^{2x}}]' = [e^{2x \cdot ln(3x+4)}]'$

$= e^{2x \cdot \ln(3x+4)} [2x \cdot \ln(3x+4)]'$

$= e^{2x \ln(3x+4)} [2 \ln(3x+4) + 2x(\frac{1}{3x+4})3]$

$$= \cancel{e}^{\cancel{\ln}(3x+4)^{2x}} \left[\quad \right]$$

$$= (3x+4)^{2x}\left[2\ln(3x+4) + \frac{6x}{3x+4}\right]$$

3.10

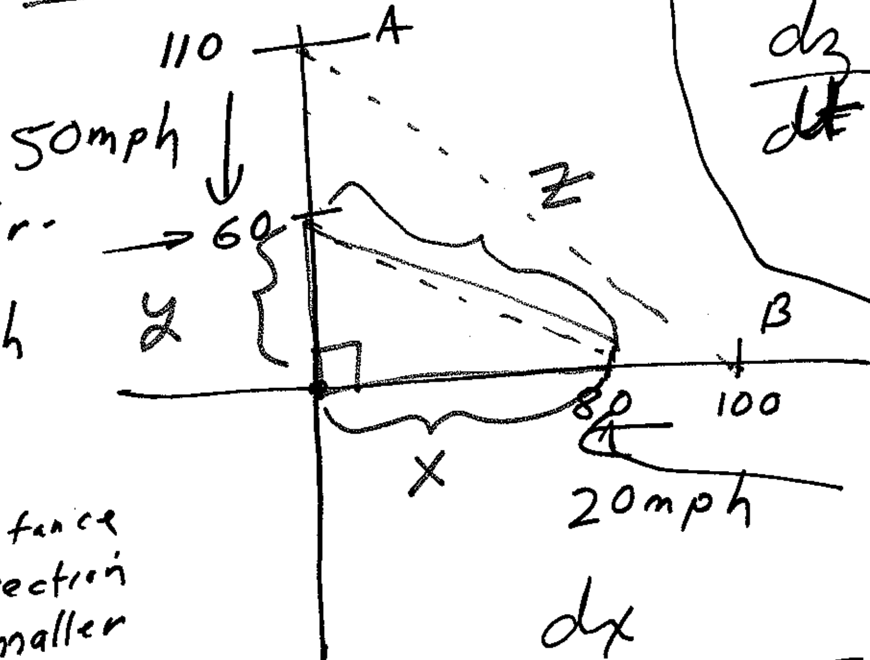
1.) Suppose car A is 110 miles north of an intersection and traveling south at 50 mph. Suppose car B is 100 miles east of the same intersection and traveling west at 20 mph. 1a.) At what rate are the cars approaching each other after 1 hour? 1b.) After 3 hours?

N
W E
S

$$\frac{dz}{dt} = ?$$

a) after 1 hr
b) after 3 hrs

after 1hr



$\frac{dy}{dt} = -50\text{mph}$

since distance to intersection getting smaller

after 1 hr

$$\frac{dx}{dt} = -20 \text{ mph}$$

$$\boxed{x^2 + y^2 = z^2}$$

$$\frac{d}{dt}\left(x^2 + y^2\right) = \frac{d\left(z^2\right)}{dt}$$

$$\cancel{2}x \cdot \frac{dx}{dt} + \cancel{2}y\frac{dy}{dt} = \cancel{2}z\frac{dz}{dt}$$

$$\frac{dz}{dt} = \frac{1}{z}\left(x\frac{dx}{dt} + y\frac{dy}{dt}\right)$$

1a) After 1 hr:

$x = 80$ $\quad (100 - 20 = 80)$

started $\quad$ went 20 miles in 1 hr

$y = 60$ $\quad (110 - 50 = 60)$

$z = 100$ $\quad (z^2 = x^2 + y^2$

$z = \sqrt{x^2+y^2} = \sqrt{80^2+60^2}$

$= \sqrt{6400+3600}$

$= \sqrt{10000}$

$= 100$

$\frac{dx}{dt} = -20$ mph

$\frac{dy}{dt} = -50$ mph

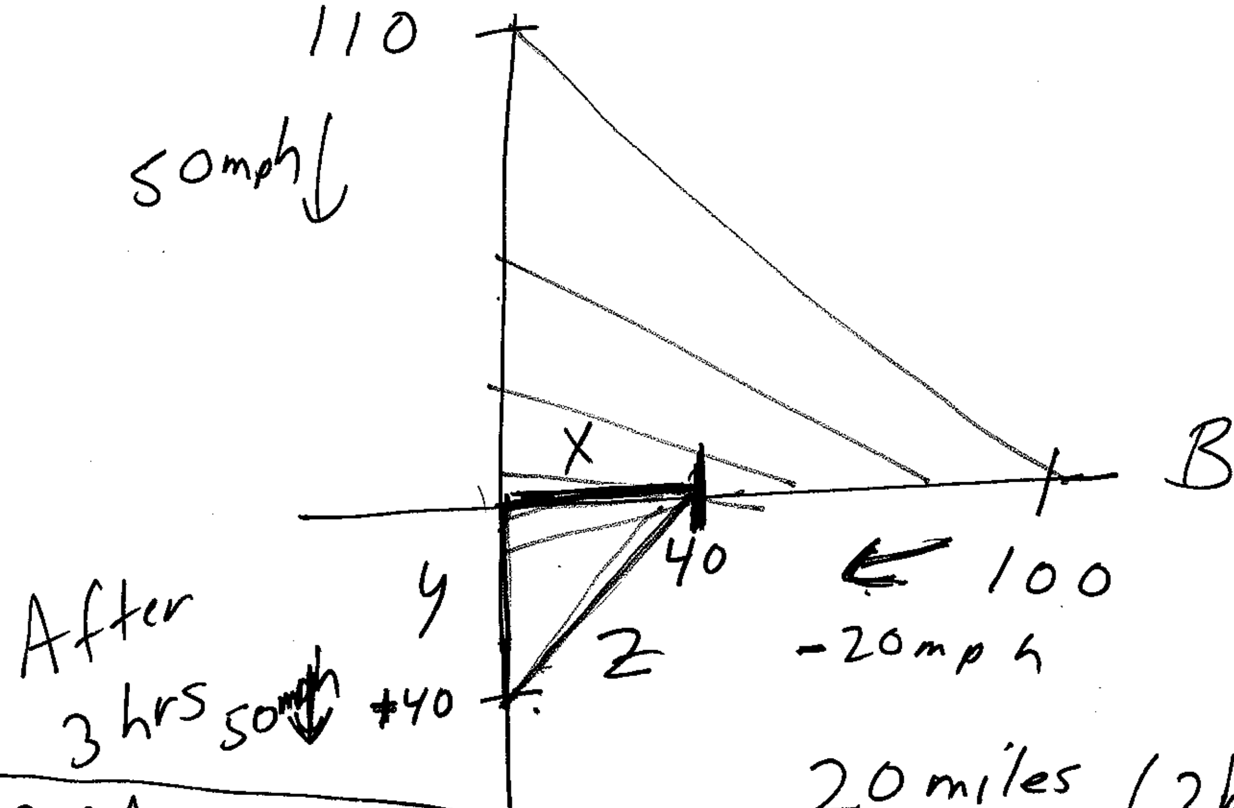
$$\frac{dz}{dt} = \frac{1}{100}\left(80(-20) + 60(-50)\right)$$

$$= \frac{-16\cancel{00} + -30\cancel{00}}{1\cancel{00}} = -46 \text{ mph}$$

getting closer

The cars are getting <u>closer</u> at a rate of 46 mph

N
W E
S



car A

$(50\,mph)(3) = 150$

$110 - 150 = 40$

$\frac{20\,miles}{\cancel{hr}}(3\,\cancel{hrs}) = 60\,miles$

$100 - 60 = 40\,miles$

$$x^2 + y^2 = z^2$$

$$\frac{dz}{dt} = \frac{1}{z}\left(x\frac{dx}{dt} + y\frac{dy}{dt}\right)$$

After 3 hrs

$x = 40$

$y = {}^{+}40$ since negative sign means getting closer

$$z = \sqrt{x^2 + y^2} = \sqrt{40^2 + 40^2} = \sqrt{2(40)^2} = 40\sqrt{2}$$

$$\frac{dx}{dt} = -20\,mph \qquad \frac{dy}{dt} = {}^{+}50\,mph$$

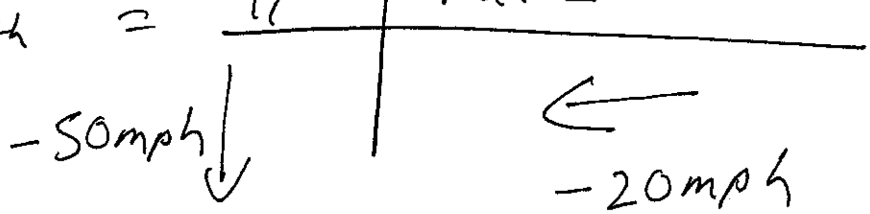
$$\frac{dz}{dt} = \frac{1}{40\sqrt{2}}\left(\underbrace{40(-20)}_{\text{getting closer to intersection}} + \underbrace{40(50)}_{\text{getting further from intersection}}\right)$$

$$= \frac{30}{\sqrt{2}} \text{ mph}$$

Cars are moving apart at a rate of $\frac{30}{\sqrt{2}}$ mph

Alternatively,
could choose negative sign to mean something else

could choose going east = negative rate
going south = " rate



After 3hrs

$x = 40$

$y = -40$

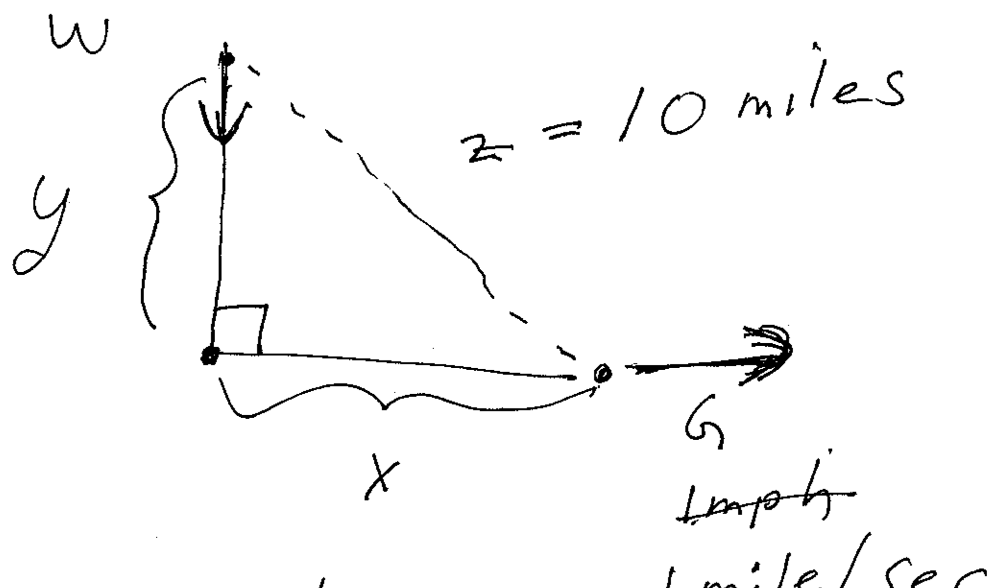
$z = 40\sqrt{2}$

$\frac{dx}{dt} = -20$

$\frac{dy}{dt} = -50$

2.) Suppose the distance between two planes must be maintained at 10 miles. Suppose plane W is north of a radio tower and moving south while plane G is east of the same radio tower. If plane G is moving east at 1 mile/second, how fast should plane W be moving when plane G is 6 miles from the radio tower?

$\boxed{\frac{dy}{dt} = ?}$ when plane G is 6 miles from radio tower



$$\frac{dx}{dt} = 1$$

$$x^2 + y^2 = z^2$$

$$x^2 + y^2 = 10^2$$

$$\frac{d}{dt}(x^2 + y^2) = \frac{d}{dt}(100)$$

$$\cancel{2}x \cdot \frac{dx}{dt} + \cancel{2}y \cdot \frac{dy}{dt} = 0 \Rightarrow \frac{dy}{dt} = \frac{-x}{y}\frac{dx}{dt}$$

$$\frac{dy}{dt} = \frac{-x}{y}\frac{dx}{dt}$$

$x = 6$ $\qquad$ $\frac{dx}{dt} = 1$

$y =$

$x^2 + y^2 = z^2$

$6^2 + y^2 = 10^2 \Rightarrow y = 8$

$$\frac{dy}{dt} = -\frac{6}{8}(1) = \frac{-3}{4}$$ ~~mph~~ miles/second