

*Increasing/Decreasing Test:*

If  $f'(x) > 0$  for all  $x \in (a, b)$ , then  $f$  is increasing on  $(a, b)$

If  $f'(x) < 0$  for all  $x \in (a, b)$ , then  $f$  is decreasing on  $(a, b)$

*First derivative test:*

Suppose  $c$  is a critical number of a continuous function  $f$ , then

Defn:  $f$  is **concave down** if the graph of  $f$  lies below the tangent lines to  $f$ .

Defn:  $f$  is **concave up** if the graph of  $f$  lies above the tangent lines to  $f$ .

*Concavity Test:*

If  $f''(x) > 0$  for all  $x \in (a, b)$ , then  $f$  is concave upward on  $(a, b)$ .

If  $f''(x) < 0$  for all  $x \in (a, b)$ , then  $f$  is concave down on  $(a, b)$ .

Defn: The point  $(x_0, y_0)$  is an **inflection point** if  $f$  is continuous at  $x_0$  and if the concavity changes at  $x_0$

*Second derivative test:* If  $f''$  continuous at  $c$ , then

If  $f'(c) = 0$  and  $f''(c) > 0$ , then  $f$  has a local minimum at  $c$ .

If  $f'(c) = 0$  and  $f''(c) < 0$ , then  $f$  has a local maximum at  $c$ .

If  $f'(c) = 0$  and  $f''(c) = 0$ , second derivative test gives no info.

Converses are not true:

Increasing/Decreasing Test

If  $f'(x) > 0$  for all  $x \in (a, b)$ , then  $f$  is increasing on  $(a, b)$

$f$  increasing on  $(a, b)$  does not imply  $f'(x) > 0$  for all  $x \in (a, b)$ .

Ex:

If  $f'(x) < 0$  for all  $x \in (a, b)$ , then  $f$  is decreasing on  $(a, b)$

$f$  decreasing on  $(a, b)$  does not imply  $f'(x) < 0$  for all  $x \in (a, b)$ .

Ex:

*Concavity Test:*

If  $f''(x) > 0$  for all  $x \in (a, b)$ , then  $f$  is concave upward on  $(a, b)$ .

$f$  concave upward on  $(a, b)$  does not imply  $f''(x) > 0$  for all  $x \in (a, b)$ .

Ex:

If  $f''(x) < 0$  for all  $x \in (a, b)$ , then  $f$  is concave down on  $(a, b)$ .

$f$  concave downward on  $(a, b)$  does not imply  $f''(x) < 0$  for all  $x \in (a, b)$ .

Ex: