

Thm 2.8: An acyclic digraph, D has a unique vertex basis consisting of all vertices with no incoming arcs.

Proof: Let B be the set of all vertices with no incoming arcs. If $v \in B$. Then since v has no incoming arcs, v is only reachable from itself. Thus B must be a subset of any vertex base. Suppose $u \notin B$. Let $u_1 = u$. Then u_1 has an incoming arc (u_2, u_1) . If $u_2 \in B$, then u_1 is reachable from a vertex in B . If $u_2 \notin B$ then u_2 has an incoming arc (u_3, u_2) .

Suppose the path u_n, \dots, u_1 is defined such that all vertices are distinct.

If $u_n \in B$, then u is reachable from a vertex in B .

If $u_n \notin B$ then u_n has an incoming arc (u_{n+1}, u_n) . If $u_{n+1} = u_i$ for some $i = 1, \dots, n$, then u_{n+1}, u_n, \dots, u_i is a cycle, a contradiction. Hence all the vertices of u_{n+1}, u_n, \dots, u_1 are distinct.

Since the number of vertices of D is finite, this process must eventually end, say with the path u_t, \dots, u_1 . Since we cannot continue this process, u_t must not have any incoming arcs. Hence $u_t \in B$, and hence u is reachable from a vertex in B . Thus any vertex basis must be contained in B . Hence B is the unique vertex basis of D .