Math 150 Exam 2

November 3, 2006

[10] 1a.) What is the coefficient of $x^3y^2z^5$ in the expansion of $(2x+y-z)^{10}$:

$(2^3)(-1)^5(\frac{10!}{3!2!5!}) = \frac{-8(10!)}{3!2!5!}$

[6] 1b.) What is the coefficient of $x^3y^2z^4$ in the expansion of $(2x+y-z)^{10}$: $\underline{0}$

[84] Choose 4 from the following 5 problems. Circle your choices: A B C D E
You may do all 5 problems in which case your unchosen problem can replace your lowest problem at 4/5 the value. Note you must fully explain your answers.

A.) Use Newtons binomial theorem to estimate $\sqrt{5}$ (expand to at least 4 terms).

$$\sqrt{5} = (1+4)^{\frac{1}{2}} = 2(\frac{1}{4}+1)^{\frac{1}{2}} = 2\Sigma_{k=0}^{\infty}\binom{\frac{1}{2}}{k}(\frac{1}{4})^k \sim 2[1+(\frac{\frac{1}{2}}{1!})(\frac{1}{4})+\frac{(\frac{1}{2})(\frac{-1}{2})}{2!}(\frac{1}{4})^2+\frac{(\frac{1}{2})(\frac{-1}{2})(\frac{-3}{2})}{3!}(\frac{1}{4})^3]$$
$$= 2[1+\frac{1}{8}-\frac{1}{128}+\frac{1}{16}(\frac{1}{64})] = 2+\frac{1}{4}-\frac{1}{64}+\frac{1}{8}(\frac{1}{64}) = 2+\frac{1}{4}-\frac{1}{64}+\frac{1}{512}$$

B.) Find the number of integers between 1 and 10,000 inclusive that are not divisible by 4, 6, 10.

Similar to ch 6: 2

C.) What is the number of ways to place ten nonattacking rooks on the 10-by-10 board with forbidden positions as shown?

Note you can number the columns and rows any way you want.

$Let A = \{(1,1),(2,1),(2,2)\}$.

$Let B = \{(3,3),(4,3)\}$.

Let r_1 = number of ways to place one rook in a forbidden position = number of forbidden positions = 5.

if 1 rook in A: 3

if 1 rook in B: 2

Let r_2 = number of ways to place two rooks in a forbidden positions: 6

if 2 rooks in A: 1

if 1 rook in A, 1 in B: 3 + 2 = 5

if 2 rooks in B: 0

Let r_3 = number of ways to place three rooks in a forbidden positions: 2

if 3 rooks in A: 0

if 2 rooks in A, 1 in B: 2

if 1 rook in A, 2 in B: 0

if 3 rooks in B: 0

$r_i = 0$ for $i > 3$

Hence by thm 6.4.1, the number of different assignments is

$10! - r_1 9! + r_2 8! - r_3 7! = 10! - 5(9!) + 6(8!) - 2(7!)$

D.) Let R_n denote the number of permutations of $X_n = \{1, 2, ..., n\}$, $n \geq 3$ in which neither the pattern 12 nor the pattern 23 occurs (note there are only 2 restrictions, for example, the pattern 34 may or may not occur). Determine a formula for R_n and prove your formula is correct.

$n! - 2(n-1)! + (n-2)!$

E.) Consider the partially ordered set $(\mathcal{P}(X_2), \subset)$ of subsets of $\{1, 2\}$ partially ordered by containment. Let a function f in $\mathcal{F}(\mathcal{P}(X_2))$ be defined by

$$f(A, B) = \begin{cases} 2 & \text{if } A = B \\ 3 & \text{if } A \subset B,\ A \neq B \\ 0 & \text{otherwise} \end{cases}$$

Find the following:

$f^{-1}(\emptyset, \emptyset) = \underline{2}$ $\qquad$ $f^{-1}(\emptyset, \{1\}) = \underline{3}$ $\qquad$ $f^{-1}(\emptyset, \{2\}) = \underline{3}$

$f^{-1}(\emptyset, \{1, 2\}) = \underline{3}$ $\qquad$ $(f * f)(\emptyset, \{1\}) = \underline{section 6.6}$