Math 150 Final Exam
December 13, 2006

Choose 7 from the following 10 problems. Circle your choices: 1 2 3 4 5 6 7 8 9 10
You may do more than 7 problems in which case your unchosen problems can replace your lowest one or two problems at 2/3 the value as discussed in class.

1.) Show that every sequence $a_1, a_2, ..., a_{n^2+1}$ contains either an increasing or decreasing subsequence of length $n + 1$.

2.) The Ramsey number $r(3, 3) =$ _______________. Prove your answer.

3.) Is the intersection $R \cap S$ of two equivalence relations R and S on a set X always an equivalence relation on X? Is the union $R \cup S$ of two equivalence relations R and S on a set X always an equivalence relation on X? Prove your answer.

4.) Find the number of integral solutions to the equation $x_1 + x_2 + x_3 + x_4 = 60$ such that $0 \leq x_1 \leq 10$, $1 \leq x_2 \leq 5$, $x_3 \geq -2$, and $x_4 \geq 4$.

5.) Let D_n be the number of derangements of $\{1, 2, ..., n\}$. Determine a formula for D_n. Prove your answer.

6.) Solve the recurrence relation $h_n = 2h_{n-1} + 3^n$ with initial value $h_0 = 4$

7a.) Determine the generating function for the number h_n of n-combinations of fruit consisting of apples, oranges, bananas, pears, and kiwis in which there are an odd number of apples, the number of oranges is a multiple of 4, the number of bananas is at most 3, the number of pears is 0 or 1, and there are at least 2 kiwis.

7b.) Find a formula for h_n.

8a.) Find the number of partitions of 6 distinguishable objects into 3 nonempty distinguishable boxes.

8b.) Find the difference table for $h_n = n^2 + 1$

8c.) $\Sigma_{k=0}^{n} h_k =$ _______________

9a.) Find the number of subsets of $\{1, 2, 3, ..., 10\}$.

9b.) Find the number of subsets of $\{1, 2, 3, ..., 10\}$ which have exactly 8 elements .

9c.) Find the number of permutations of $\{1, 2, 3, ..., 10\}$ which have exactly 8 elements.

9d.) Find the number of permutations of $\{3 \cdot a, 4 \cdot b, 1 \cdot c\}$ which have exactly 8 elements.

9e.) Find the number of partitions of 25 indistinguishable objects into 10 distinguishable boxes.

10a.) Expand $(x - 2y)^6$ using the binomial theorem.

10b.) What is the coefficient of $x^4y^3z^2$ in the expansion of $(x - y + 3z)^9$: ________________

10c.) What is the coefficient of $x^3y^3z^2$ in the expansion of $(x - y + 3z)^9$: ________________

10d.) The inversion sequence for the permutation 615423 is ________________________

10e.) The permutation corresponding to the inversion sequence 5, 1, 3, 2, 1, 0 is ______________________