3.1 Basic Counting

A *partition* of a set S is a collection of subsets S_i of S such that $S = \cup S_i$ and $S_i \cap S_j = \emptyset$ for all $i \neq j$.

Addition Principle: If $S = S_1 \cup S_2$ and $S_1 \cap S_2 = \emptyset$, then $|S| = |S_1| + |S_2|$.

If $S_1 \cap S_2 = \emptyset$ and if $x \in S$ implies $x \in S_1$ OR $x \in S_2$, then $|S| = |S_1| + |S_2|$.

Multiplication Principle: If $S = S_1 \times S_2$, then $|S| = |S_1||S_2|$.

$x = (a, b) \in S$ implies $a \in S_1$ AND $b \in S_2$, then $|S| = |S_1||S_2|$.

Subtraction Principle: Suppose $A \subset U$. Let the complement of A in $U = \overline{A} = \{x \in U \mid x \notin A\}$. Then $|A| = |U| - |\overline{A}|$.

Division Principle: Suppose $S = \cup_{i=1}^{k} S_i$. If $|S_i| = n$, then $k = \frac{|S|}{n}$.

Counting Problems:

1.) Order matters (ordered arrangements or selections)

1a.) no repeats allowed

1b.) (limited) repeats allowed

2.) Order does not matter (unordered arrangements or selections)

2a.) no repeats allowed

2b.) (limited) repeats allowed

Defn: A *multiset* is a collection of objects were repeats are allowed.

Set: $\{a, a, b, b, b\} = \{a, b\}$

Multiset: $\{a, a, b, b, b\} = \{2 \cdot a, 3 \cdot b\}$

Subsets: Suppose a set B has n elements (i.e., $|B| = n$). The number of subsets of B is

Suppose a symbol can be either a number between 0 and 9 or a letter. How many are symbols there?

How many sequences consisting of one letter followed by one single digit number (0 - 9) are possible?

How many different license plates are possible if 3 letters followed by 3 numbers are used?

How many different license plates are possible if 3 letters followed by 3 numbers are used and the license plate starts with a vowel if and only if the plate contains exactly one vowel?

Subsets

Suppose a set A has four elements (i.e., the cardinality of $A = |A| = 4$)

The number of subsets of A is

The number of nonempty subsets of A is

A pizza parlor offers 4 different toppings (sausage, onions, chicken, walnuts). How many different types of pizzas can one order?

Suppose a set B has n elements (i.e., $|B| = n$). The number of subsets of B is

Example: How many 10-digit telephone numbers are there if

1.) there are no restrictions.

2.) the digits must all be distinct.

3.) The area code cannot begin with a 0 or 1 and must have a 0 or 1 in the middle.

Example: How many different seven-digit numbers can be constructed out of the digits 2, 4, 8, 8, 8, 8, 8?

Example: How many different seven-digit numbers can be constructed out of the digits 2, 2, 8, 8, 8, 8, 8?

Example A: How many numbers between 100 and 1000 have distinct digits.

Example B: How many odd numbers between 100 and 1000 have distinct digits.

Example C: How many even numbers between 100 and 1000 have distinct digits.

method 1:

method 2:

2.2 Permutations:

Suppose $|S| = n$.

An r-*permutation* of S is an ordered arrangement of r of the n elements of S.

If $r = n$, then an r-permutation of S is a *permutation* of S.

$P(n, r) =$ number of r-permutations of S where $|S| = n$.

4 TA's need to be assigned to 4 different classes. How many different possible assignments are there?

4 classes need to be assigned a TA. There are 10 TAs. How many different possible assignments are there?

If $r > n$, then $P(n, r) =$

$P(0, 0) =$ $\quad$ $P(n, 0) =$ $\quad$ $P(n, 1) =$ $\quad$ $P(n, n) =$

$n! = n(n-1)(n-2)...(2)(1)$

$0! = 1$

Thm 2.2.1: If $r \leq n$, then $P(n, r) = \frac{n!}{(n-r)!}$

2.3 Combinations

An r-*combination* of S is an r-element subset of S (ORDER DOES NOT MATTER).

$C(n, r) =$ number of r-combinations of S where $|S| = n$.

How many different math teams consisting of 4 people can be formed if there are 10 students from which to choose?

Thm: $C(n, r) = \begin{pmatrix} n \\ r \end{pmatrix} = \frac{n!}{r!(n-r)!} = \frac{P(n,r)}{r!}$

Cor: $C(n, r) = C(n, n - r)$

Cor: $C(n, r) = C(n - 1, r - 1) + C(n - 1, r)$

Cor: Pascal's Triangle.

Cor: $\Sigma_{i=0}^{n} \begin{pmatrix} n \\ i \end{pmatrix} = 2^n$

How many different proteins containing 10 amino acids can be formed if the protein contains 5 alanines(A), 3 leucines (L), and 2 serines (S)?