22M:132: Topology Exam 2
Nov. 18, 2010

[10] 1.) Definition: X is locally path connected if

2.) Suppose $\mathbb{R}^\omega$ has the uniform topology with uniform metric $\overline{\rho}$.

[4] 2a.) $\overline{\rho}(\mathbf{x}, \mathbf{y}) =$ ______________________________

[4] 2b.) $B_{\overline{\rho}}(\mathbf{0}, 2) =$ ______________________________

[4] 2c.) Let $\mathbf{x}_n = (\frac{1}{n}, \frac{1}{n}, \frac{1}{n}, ...) \in \mathbb{R}^\omega$. Let $A = \{\mathbf{x}_i \mid i \in \mathbb{Z}_+\}$. Then $A^o =$ __________________ .

[18] 3.) Circle T for true and F for false. If a statement is false, show that the statement is false by providing a counter-example. You do not need to prove that your example is a counter-example.

3a.) If A is a compact subspace of X, then A is closed in X.

T F

3b.) Let A be a connected subspace of X. If $A \subset B \subset \overline{A}$, then B is connected.

T F

3c.) Let A be a path connected subspace of X. If $A \subset B \subset \overline{A}$, then B is path connected.

T F

[60] Prove 2 of the following 5. Clearly indicate your choices. You may do a third problem for extra credit.

First two choices: ____________

Third choice (extra credit): ____________

1. Compact Hausdorff implies T_3.

2. Define an equivalence relation on $\mathbb{R}^1$ by $x \sim y$ if $x - y \in \mathbb{Z}$. Let $X/\sim$ be the corresponding quotient space. It is homeomorphic to a familiar space. What is it? [Hint: set $g(x) = e^{2\pi x}$]

3. Let H be a subspace of the topological group $(G, \cdot)$. Show that if H is also a subgroup of G, then both H and $\overline{H}$ are topological groups. Hint: Recall that H is a subgroup of the group G if and only if it is nonempty and closed under products and inverses.

4. Let $\mathbf{x}_n = (\frac{1}{n}, \frac{1}{n}, \frac{1}{n}, ...) \in \mathbb{R}^\omega$. Let $A = \{\mathbf{x}_i \mid i \in \mathbb{Z}_+\}$. If $\mathbb{R}^\omega$ has the uniform topology, determine $\overline{A}$.

5. Suppose X is locally compact and $f : X \to Y$ is a continuous, surjective, open map. Then $f(X)$ is locally compact.