22M:132: Topology Exam 1

Sept 28, 2010

[10] 1.) Define: Basis for a topology

[12] 2.) Suppose $\mathbb{R}$ = the set of real numbers has the standard topology. Calculate the following **in** $(0, \infty)$ **where** $(0, \infty) \subset \mathbb{R}$ **has the subspace topology:**

$(0, 1)^o =$ ____________ $\overline{(0, 1)} =$ ____________ $(0, 1)' =$ ____________

3.) The following two statements are false. Show that the statements are false by providing counter-examples. Very briefly explain your counter-examples.

[9] 3a.) Suppose $A \subset Y \subset X$. Let A^o denote the interior of A in X and let $Int_Y A$ = interior of A in Y. Then $Int_Y A = A^o \cap Y$.

[9] 3b.) $\overline{A \cap B} = \overline{A} \cap \overline{B}$.

[60] Prove 2 of the following 4. **Clearly indicate your TWO choices.** You may do more than 2 problems in which case I may substitute one of your unchosen problems for one of your two choices (with a penalty) if it improves your grade.

(1) $(2\mathbb{Z}, +)$ is a topological group where $(2\mathbb{Z}, +)$ is the even integers with the operation of addition.

(2) $f_i : X_i \to Y_i$ continuous implies $F : X_1 \times X_2 \to Y_1 \times Y_2$, $F(x_1, x_2) = (f_1(x_1), f_2(x_2))$ is continuous.

(3) Let $\mathbb{R}$ have the topology $T_{ray} = \{(r, \infty) \mid r \in \mathbb{R}\}$. Find $\overline{(0,1)}$ in this topology. Fully prove.

(4) Suppose T_α is a topology for all $\alpha \in A$
a.) Prove or disprove: $\cup_{\alpha \in A} T_\alpha$ is a topology.
b.) Prove or disprove: $\cap_{\alpha \in A} T_\alpha$ is a topology.