$\mathcal{B} = \{(a, b) \mid a, b \in \mathcal{R}, a < b\}$ is NOT a topology.

Pf 1: $\frac{a+b}{2} \in (a, b)$. Thus $(a, b) \neq \emptyset$. Hence $\emptyset \notin \mathcal{B}$.

Pf 2: $b + 1 \notin (a, b)$. Thus $(a, b) \neq \mathcal{R}$. Hence $\mathcal{R} \notin \mathcal{B}$.

Pf 3: $(1, 2) \cup (3, 4) \notin \mathcal{B}$.

Pf 4: $(1, 2) \cap (3, 4) = \emptyset \notin \mathcal{B}$.

$\mathcal{T} = \{U \mid x \in U \implies \exists (a, b)\ s.t.\ x \in (a, b) \subset U\}$ is a topology.

Pf: (1) $\emptyset \in \mathcal{T}$ (vacuously true).

$\mathcal{R} \in \mathcal{T}$ since if $x \in \mathcal{R}$, then $x \in (x - 1, x + 1) \subset \mathcal{R}$.

(2) Suppose $U_\alpha \in \mathcal{T}\ \forall \alpha \in A$.

Claim: $\cup_{\alpha \in A} U_\alpha \in T$.

Let $x \in \cup_{\alpha \in A} U_\alpha$. Then