22M:100 (MATH:3600:0001) Exam 2
April 27, 2013

1.) Circle T for True and F for false.

[4] 1a.) If $y = f(t)$ and $y = g(t)$ are solutions to $y'' + p(t)y' + q(t)y = 0$, then $y = c_1 f(t) + c_2 g(t)$ is also a solution to this differential equation for any constants c_1 and c_2.

T F

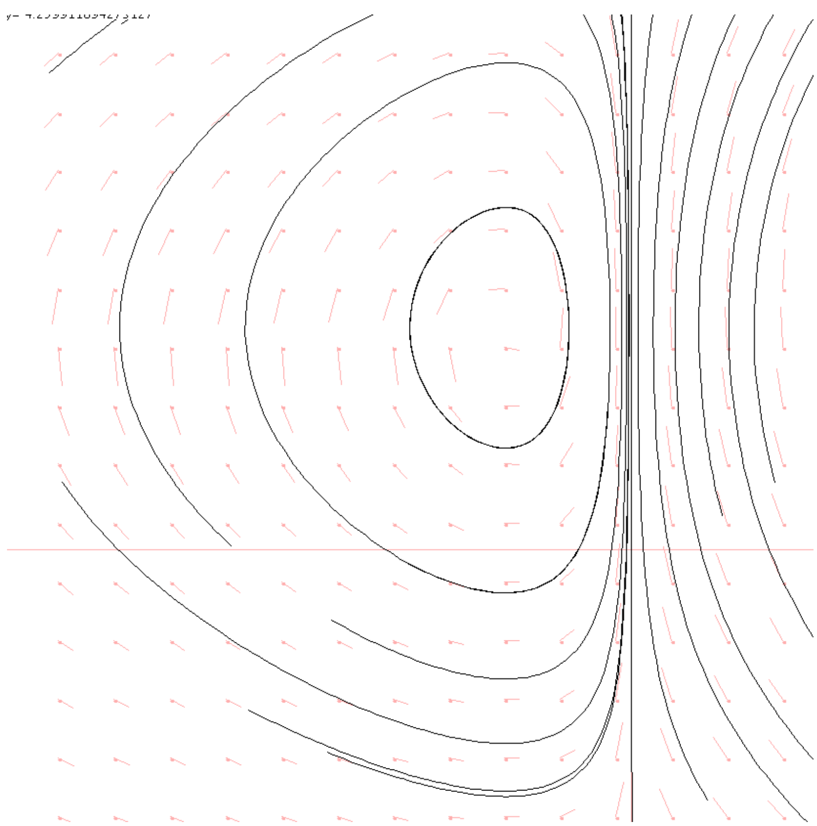
[4] 1b.) If $\mathbf{x} = \mathbf{f}(t)$ and $\mathbf{x} = \mathbf{g}(t)$ are solutions to $\mathbf{x}' = A\mathbf{x}$, then $\mathbf{x} = c_1\mathbf{f}(t) + c_2\mathbf{g}(t)$ is also a solution to this system of differential equation for any constants c_1 and c_2.

T F

[12] 2.) The phase portrait for $\frac{dx}{dt} = x(y-2)$ and $\frac{dy}{dt} = x + 3$ is drawn below. Find all equilibrium solutions and determine whether the critical point is asymptotically stable, stable, or unstable. Also classify it as to type (nodal source, nodal sink, saddle point, spiral source, spiral sink, center).

Equilibrium solution: ______________________________

Stability : ______________________________ Type: ______________________________



[20] 3.) Find a recursive formula for the constants of the series solution to $y'' + 2y = 0$ near $x_0 = 0$.

Answer: ____________________________________

[20] 4.) Suppose that $a_{2k} = \dfrac{-a_{2k-2}}{4k^2}$. Use induction to prove that $a_{2k} = \dfrac{(-1)^k a_0}{4^k (k!)^2}$ for all $k \geq 0$.

[20] 5.) Solve $\mathbf{x}' = \begin{pmatrix} 0 & 3 \\ -3 & 0 \end{pmatrix} \mathbf{x}$

Answer: ____

If you forgot the formula, you can guess it, but also draw a rough sketch of the phase portrait for partial credit (not needed if your answer is correct).

[20] 6.) Suppose the matrix A has eigenvalue -1 with eigenvector $\begin{pmatrix} 0 \\ 3 \end{pmatrix}$ and eigenvalue 4 with eigenvector $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$. Draw the phase portrait for this system of differential equations. Identify the equilibrium solution. Also state the general solution.

Equilibrium solution is ______________________________________

General solution is ______________________________________