22M:100 (MATH:3600:0001) Quiz 3
April 16, 2013

[20] 1.) Find all singular points of the given equation and determine whether each one is regular or irregular.

1a.) $x^2y'' + 2y' + 3xy = 0$

Singular points: $x = 0$

irregular

$\lim_{x\to 0} x\left(\frac{2}{x^2}\right) = \lim_{x\to 0} \frac{2}{x} = \infty$

1b.) $(x+1)y'' + 3xy' + (x+2)y = 0$

singular points: $x = -1$

regular

$\lim_{x\to -1} (x+1)\left(\frac{3x}{x+1}\right) = -3$

$\lim_{x\to -1} (x+1)\left(\frac{x+2}{x+1}\right) = 1$

[30] 2.) Find the largest possible domain for $f(x) = \sum_{n=0}^{\infty} \frac{nx^n}{4^n}$

$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \left|\frac{(n+1)x^{n+1}}{4^{n+1}} \cdot \frac{4^n}{nx^n}\right|$

$= \lim_{n\to\infty} \left|\frac{(n+1)x}{4n}\right| = \frac{|x|}{4} \lim_{n\to\infty} \left|\frac{n+1}{n}\right| = \frac{|x|}{4} < 1$

$|x| < 4$

$(-4, 4)$

end pts are not in the domain

$f(4) = \sum_{n=0}^{\infty} n$

$f(-4) = \sum_{n=0}^{\infty} n(-1)^n$

[50] 3.) Solve $\mathbf{x}' = \begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix}\mathbf{x}$

$\begin{vmatrix} 2-\lambda & 0 \\ 0 & -\lambda \end{vmatrix} = -2\lambda + \lambda^2 + 10$

$\lambda(\lambda - 2) = 0$

$\lambda = 0, 2$ +10

$\lambda = 0$: $\begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix} \rightarrow 2x_1 = 0 \Rightarrow x_1 = 0$, $x_2 = \text{free}$ $\quad \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ +10

$\lambda = 2$: $\begin{bmatrix} 0 & 0 \\ 0 & -2 \end{bmatrix} \rightarrow -2x_2 = 0 \Rightarrow x_2 = 0$, $x_1 = \text{free}$ $\quad \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ +10

$$\mathbf{x} = c_1 \begin{bmatrix} 0 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ 0 \end{bmatrix} e^{2t}$$ +10