22m:033 Notes:
5.3 Diagonalization, an Example

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April 22, 2010

1 Diagonalization and linear transformations

Example 1.1 We leave it as an exercise to verify the following calculations:

Let $A = \begin{pmatrix} -4 & -6 \\ 3 & 5 \end{pmatrix}$ and let $P = \begin{pmatrix} -1 & -2 \\ 1 & 1 \end{pmatrix}$.

We can calculate that $P^{-1} = \begin{pmatrix} 1 & 2 \\ -1 & -1 \end{pmatrix}$.

We can easily check that the columns of P are eigenvectors for A and in fact the first column has eigenvalue 2 and the second column has eigenvalue -1.

Also we can easily check that $D = P^{-1}AP = \begin{pmatrix} 2 & 0 \\ 0 & -1 \end{pmatrix}$. (Recall, we could also write this as $A = PDP^{-1}$.)

Graphically we can view this as a diagram:

$$\begin{array}{ccccc} R^2 & - & D & \rightarrow & R^2 \\ | & & & & \uparrow \\ P & & & & P^{-1} \\ \downarrow & & & & | \\ R^2 & - & A & \rightarrow & R^2 \end{array} \quad \text{or} \quad \begin{array}{ccccc} R^2 & - & D & \rightarrow & R^2 \\ | & & & & | \\ P & & & & P \\ \downarrow & & & & \downarrow \\ R^2 & - & A & \rightarrow & R^2 \end{array}$$

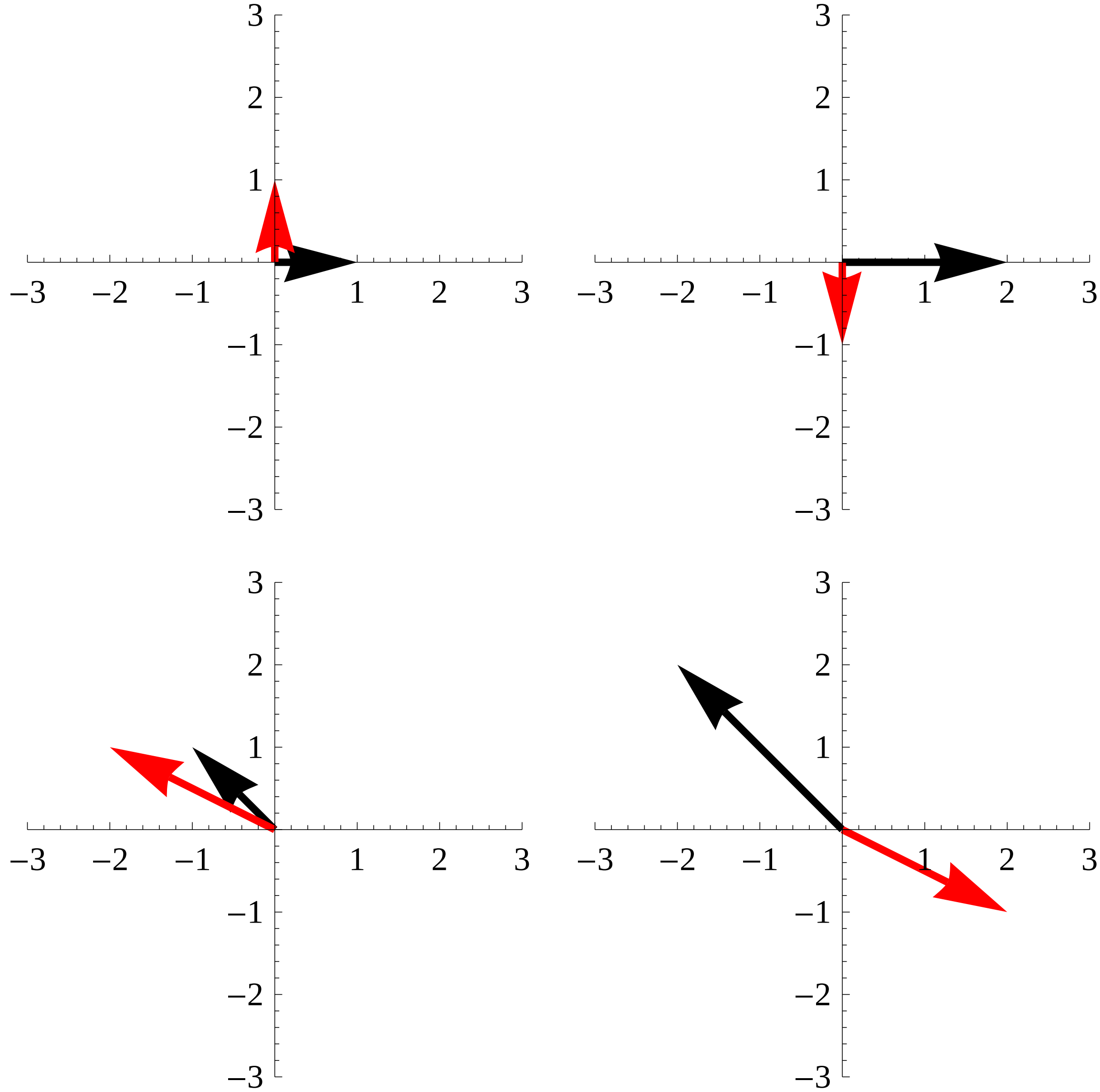

Figure 1: Upper left is standard basis. Top row across is transformation given by D. Lower left is eigenbasis for A. Bottom row is transformation given by A. From top to bottom (both on right and left) transformation is given by P.