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**Endomorphisms of $\mathcal{B}(\mathcal{H})$. II. Finitely correlated states on $\mathcal{O}_n$. (English. English summary)**

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Let $\mathcal{B}(\mathcal{H})$ denote the algebra of all bounded linear operators on a separable, infinite-dimensional, complex Hilbert space $\mathcal{H}$. If $\alpha\colon \mathcal{B}(\mathcal{H}) \to \mathcal{B}(\mathcal{H})$ is a unital endomorphism, the authors say that $\alpha$ is ergodic [resp., a shift] if $\{T \in \mathcal{B}(\mathcal{H})\colon\ \alpha(T) = T\}$ [resp., $\bigcap_{n=1}^{\infty} \alpha^n[\mathcal{B}(\mathcal{H})]$ ] consists only of scalar multiples of the identity operator. The (Powers) index of $\alpha$ is defined to be the extended integer $n$ such that $\alpha[\mathcal{B}(\mathcal{H})]' \bigcap \mathcal{B}(\mathcal{H})$ is isomorphic to the factor of type $\mathrm{I}_n$. Two unital endomorphisms $\alpha$ and $\beta$ are conjugate if there exists an automorphism $\gamma$ of $\mathcal{B}(\mathcal{H})$ such that $\alpha = \gamma\beta\gamma^{-1}$.

If $O_n$ denotes the Cuntz algebra of order $n$ and $\mathrm{U}(n)$ denotes the group of unitary, $n \times n$ complex matrices, then $\mathrm{U}(n)$ acts canonically on $O_n$. It is well known that there is a one-to-one correspondence between conjugacy classes of endomorphisms of $\mathcal{B}(\mathcal{H})$ of index $n$ and unitary equivalence classes of nondegenerate $*$-representations of $O_n$ in $\mathcal{B}(\mathcal{H})$ modulo the action of $\mathrm{U}(n)$. Ergodic endomorphisms correspond to irreducible representations, and hence pure states, of $O_n$, and shifts correspond to pure states such that when the corresponding GNS representation of $O_n$ is restricted to the canonical UHF subalgebra of $O_n$, the resulting image is weakly dense in the space of all operators on the representation Hilbert space. The classification problem for ergodic endomorphisms and shifts of index $n$ is thereby reduced to the classification of pure states of $O_n$ up to the action of $\mathrm{U}(n)$ and unitary equivalence.

The main result of the paper under review is the characterization and classification of those pure states $\omega$ of $O_n$ with the property that, for some nonnegative integer $k$, $\omega \circ \sigma^{k+1} = \omega \circ \sigma^k$, where $\sigma$ is the canonical shift on $O_n$. The authors show that the set $S_k$ of these states can be endowed naturally with the structure of a finite-dimensional, differentiable manifold. When an element of $S_k$ restricts to a pure state of the canonical UHF subalgebra of $O_n$, it induces a shift on $B(H)$, and the authors use the differentiable structure of $S_k$ to analyze the conjugacy classes of these shifts.

{Part I of this work has been reviewed [O. Bratteli, P. E. T. Jorgensen and G. L. Price, in Quantization, nonlinear partial differential equations, and operator algebra (Cambridge, MA, 1994), 93–138, Proc. Sympos. Pure Math., 59, Amer. Math. Soc., Providence, RI, 1996; MR 97h:46107].} *Steve Wright* (1-OAKL)