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The article continues previous work by the authors [J. Funct. Anal. 125 (1994), no. 1, 90–110; MR 95i:47067]. Here they consider affine systems in $\mathbf{R}^n$ constructed from quadruples (R, B, L, K) where R is an invertible integral matrix, B and L are finite subsets of $\mathbf{R}^n$ having the same cardinality N, and K is a lattice in $\mathbf{R}^n$. The associated affine system is defined by $\sigma_b(x) = R^{-1}x + b$ for b in B and x in $\mathbf{R}^n$. Under suitable assumptions, there exists a unique probability measure μ on $\mathbf{R}^n$ satisfying $\mu = |B|^{-1}\sum_b \mu \circ \sigma_b^{-1}$ and supported on a suitable "fractal" set $\overline{X}$. It turns out that when the matrix $U = (e^{i2\pi b\cdot l})_{b,l}$ satisfies $U^*U = UU^* = NI_N$, one gets a pair of representations (S_b) and (T_l) of the Cuntz algebra O_N acting on $L^2(\mu)$ such that $S_b^*T_l$ is a multiplication operator for all b and l. This is a substitute for the classical harmonic analysis problem which consists in finding a subset Λ of $\mathbf{R}^n$ such that the exponentials $(e^{i\lambda\cdot x})_{\lambda,\Lambda}$ form an orthonormal basis for $L^2(\mu)$. The paper also contains examples of affine systems for $N \leq 4$ including graphic illustrations of the corresponding sets $\overline{X}$. *Paul Jolissaint* (CH-NCH)