Results from MathSciNet: *Mathematical Reviews* on the Web
© Copyright American Mathematical Society 2001

2000k:35223 35P20 35B27 35K15 47D06 47F05

Bratteli, Ola (N-OSLO); **Jørgensen, Palle E. T.** (1-IA);
Robinson, Derek W. (5-ANU-MT)

Spectral asymptotics of periodic elliptic operators. (English. English summary)

Math. Z. **232** (1999), *no.* 4, 621–650.

The authors consider the C_0-semigroup $\{S_t\colon\ t \geq 0\}$ generated by a complex, second-order, strongly elliptic, periodic operator H on $\mathbf{R}^d$. They assume that H belongs to a class such that the kernel of S satisfies Gaussian bounds and is Holder continuous with bounds depending only on the ellipticity constant and the L^∞-norms of the coefficients. Sufficient conditions are that $d = 1$ or 2, or that the second-order coefficients are real or a small complex perturbation of a real matrix.

The space $L^2(\mathbf{R}^d)$ can be considered as $L^2(\mathbf{T}^d \times [0,1)^d)$ by identifying $f(x)$ with $g(z,u)$, where $g(z,u) = \sum_{n\in\mathbf{Z}^d} z^n f(u-n)$. Then S_t decomposes as $\int_{\mathbf{T}^d}^{\oplus} S_t^z\,dz$, where S^z is the semigroup on $L^2([0,1)^d)$ generated by the operator H^z corresponding to H with z-periodic boundary conditions. The authors extend the homogenization results of their earlier paper with the reviewer [J. Geom. Anal. 5 (1995), no. 4, 427–443; MR 97f:35028] to this case (with complex coefficients and lower order terms) by showing that the semigroups $S^{(m),z}$ and $S^{(m)}$, obtained by replacing x by mx in the coefficients of H, converge to the homogenized semigroups. This provides an asymptotic approximation to the eigenvalues of H^z and hence to the spectrum of H in the high frequency limit. *C. J. K. Batty* (Oxford)