
References

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- [WWW2] <http://www.electronicletters.com/papers/example/Psubsampletransform.gif>; Figure 5 in [WWW1].
- [WWW3] <http://www.electronicletters.com/papers/example/Pinversetransform.gif>; Figure 6 in [WWW1].
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Symbols

Reminder: In the symbol list and in the chapters, function spaces are defined with respect to various integrability conditions. For a function f on a space X , absolute integrability refers to $|f|$, i.e., to the absolute value of f , and to a prescribed (standard) measure on X . This measure on X is often implicitly understood, as is its σ -algebra of measurable sets. Examples: If the space X is \mathbb{R}^d , the measure will be the standard d -dimensional Lebesgue measure; for the one-torus \mathbb{T} (i.e., the circle group), it will be normalized Haar measure, and similarly for the d -torus \mathbb{T}^d ; for $X = \mathbb{Z}$, the measure will simply be counting measure; and for X_3 (the middle-third Cantor set), the measure will be the corresponding Hausdorff measure h_s of fractal dimension $s = \log_3(2)$. In each case, we introduce Hilbert spaces of L^2 -functions, and the measure will be understood to be the standard one. Same convention for the other L^p -spaces!

$A(i_1, \dots, i_n)$: the cylinder set $\{\omega \in \Omega \mid \omega_1 = i_1, \dots, \omega_n = i_n\}$, i.e., the set of infinite strings $\omega = (\omega_1, \dots)$ specified by $\omega_1 = i_1, \dots, \omega_n = i_n$ 43, 47, 85	$B(\mathcal{H})$: bounded linear operators on a Hilbert space \mathcal{H} 183, 218 \mathcal{B} : Borel σ -algebra 6, 40, 53, 204
\mathfrak{A} : the C^* -algebra of the canonical anticommutation relations 139, 140, 141, 154	\mathcal{B}_Ω : Borel σ -algebra on Ω 115
\mathfrak{A}_n : family of algebras increasing in the index n , $\{f \in C(\Omega)$ $\mid f(\omega) = f(\omega_1, \omega_2, \dots, \omega_n)\}$ 44, 45, 139	$C(\Omega)$: continuous functions on Ω 7, 27, 44, 46 CAR : canonical anticommutation relations 138–140, 154

- \mathbb{C} : the complex numbers
 25, 43, 46, 48–52, 57, 61, 140, 184,
 210, 214, 220
- \mathbb{C}^k : k -dimensional complex vector
 space
 31
- $\mathbf{C}_3, \mathbf{C}_4$: Cantor sets
 195, 197–199
- \mathcal{D} : maximal abelian subalgebra
 154
- \mathcal{D}_Y : smallest σ -algebra with respect
 to which Y is measurable
 xxiv
- \mathcal{D}_ϕ : closed linear span
 209, 217
- $e_\lambda(t) := e^{i2\pi\lambda t}, e_k(z) = z^k$: Fourier
 basis functions
 61, 71–79, 130, 192, 198
- $E_{\omega, \xi}^{(n)}, e_{\omega, \xi}^{(n)}, e(i, j), e_{i_1, \dots, i_n; j_1, \dots, j_n}^{(n)}$:
 special matrix element generators
 135, 139, 183
- \mathcal{F} : σ -algebra
 37
- \mathcal{F}_n : system of σ -algebras
 37
- GMRA : generalized multiresolution
 analysis
 114
- h : special (harmonic) function, a
 Perron–Frobenius eigenfunction
 for R_W , a measurable function on
 X such that $R_W h = h$
 xxxiv, 11, 19, 49, 55, 92, 101, 105,
 116
- h_{\min}, h_p : minimal eigenfunction for
 R_W
 100–102, 105–107
- h_3 : minimal eigenfunction corre-
 sponding to the scale-3 stretched
 Haar wavelet
 107
- h_s : Hausdorff measure
 14, 17
- \mathcal{H} : some (complex) Hilbert space
 14, 17, 114–117, 131, 136, 140,
 169–170, 180–184, 189, 190,
 196, 210, 218–219
- I : identity operator or identity matrix
 (see also $\mathbb{1}_{\mathcal{H}}$)
 115, 131, 135, 136, 139–141, 184,
 211, 214–219
- I : index set
 172, 186, 189–190
- I : multiindex
 165–168
- IFS : iterated function system
 xxxv, xlv, 5, 14, 15, 34, 35, 67, 70,
 80, 84, 99, 152, 182
- $\text{ind} \lim_{n \rightarrow \infty} \mathfrak{A}_n$: inductive limit of an
 ascending family of algebras
 139
- \mathcal{K} : some Hilbert space
 161, 169, 170, 172, 189
- ℓ^1 : all absolutely summable sequences
 66

- $\ell^2(\mathbb{N}), \ell^2(\mathbb{N}_0)$: all square-summable sequences indexed by \mathbb{N} , or by \mathbb{N}_0
31, 140, 162, 182, 190, 193, 197
- $\ell^2(\mathbb{Z})$: all square-summable sequences indexed by \mathbb{Z}
30, 32, 117, 136, 143, 191, 193, 200–202, 213, 219
- $\ell^2(X), \ell^2$: all square-summable sequences indexed by a set X or other index set
31, 66, 143, 160, 161, 168, 170, 172, 184, 189
- $L^1(\mathbb{R})$: all absolutely integrable functions on \mathbb{R}
130
- $L^2(\mathbb{R})$: all square-integrable functions on \mathbb{R}
xxxii, 4, 5, 10, 12–16, 29, 33, 65, 71, 87, 91, 103–105, 109, 112, 114, 129, 130, 158, 162–163, 165, 181, 190–194, 198
- $L^2(\mathbb{R}^d)$: all square-integrable functions on \mathbb{R}^d
4, 22, 97, 109, 142, 229, 230
- $L^2(\mathbb{T})$: all square-integrable functions on \mathbb{T}
66, 132, 136, 162, 167, 182, 190, 192, 193, 196, 197, 210, 213, 214, 219
- $L^2(\cdot)$: all square-integrable functions on some specified set with its standard measure
14, 17, 72, 77, 79, 112, 132, 136, 191, 196–198
- $L^2(X, \mathcal{B}, \mu), L^2(\mu)$: all square-integrable functions on the σ -finite measure space (X, \mathcal{B}, μ)
31, 72
- $L^\infty(\mathbb{T})$: all essentially bounded and measurable functions on \mathbb{T}
95, 163, 190, 191
- $L^\infty(X)$: all essentially bounded and measurable functions on X with respect to the standard measure and σ -algebra of measurable subsets
9, 43, 44, 49, 115
- MRA : multiresolution analysis
6, 181, 194, 198
- m : function on \mathbb{T} representing a digital filter
4, 10, 114
- m_i : multiband filter functions
123, 126, 190, 191, 194, 211
- $m_0, \boxed{m_0}$: low-pass filter
111, 124–129
- $m_1, \boxed{m_1}$: high-pass filter
111, 124–129
- M : multiplication operator
213
- $M_n = M_n(\mathbb{C})$: $n \times n$ complex matrices
139
- $M_{2^n} := M_2 \otimes \cdots \otimes M_2$
139

- \mathbb{N} : the positive integers or natural numbers
6, 11, 135
- $\mathbb{N}_0 := \{0, 1, 2, \dots\} = \{0\} \cup \mathbb{N}$
5, 11, 59, 66, 85, 116, 117, 159,
160, 164, 166, 182, 186, 188
- ONB : orthonormal basis in a Hilbert space
13, 15, 16, 56, 71, 72, 76, 77, 103,
104, 140–198 *passim*
- $\mathcal{O}_n, \mathcal{O}_N, \mathcal{O}_2$: Cuntz algebra
131, 136, 139–152, 154, 158,
161–164, 167, 170, 176, 179–184,
189, 190, 192, 194, 196, 203, 205,
211, 214, 218, 219
- P_x : transition probability initialized at x ; measure on Ω such that
 $P_x[f] = P_x^{(n)}[f]$ for all $f \in \mathfrak{A}_n$
5–11, 19, 26, 37, 43, 44, 62, 100
- $P_x(\cdot | \cdot)$: conditional probability initialized at x
51
- $P_x(\mathbb{N}_0)$: path-space measure of the natural numbers \mathbb{N}_0 as subset of Ω
 $:= \sum_{k \in \mathbb{N}_0} P_x(\{\omega(k)\})$, where
 $P_x(\{\omega(k)\}) = \prod_{p=1}^n W(\tau_{\omega_p} \cdots \tau_{\omega_1}(x)) \cdot \prod_{m=1}^{\infty} W(\tau_0^m \tau_{\omega_n} \cdots \tau_{\omega_1}(x))$
11, 18, 60, 71, 78, 86, 88–91, 100,
102, 116
- $P_x(\mathbb{Z})$: path-space measure of the integers \mathbb{Z} as subset of Ω
11, 18, 60, 64, 71, 90, 116
- $P_x^{(n)}[f]$: transition probability initialized at x and conditioned by n coordinates
 $:= \sum_{(\omega_1, \dots, \omega_n)} \prod_{p=1}^n W(\tau_{\omega_p} \cdots \tau_{\omega_1}(x)) \cdot f(\omega_1, \dots, \omega_n)$, $f \in \mathfrak{A}_n$
21, 44, 45, 63, 64, 116, 122
- $\text{Pos}(\mathcal{H})$: operator with spectrum contained in $[0, \infty)$
114, 115, 117
- R_W, R : Perron–Frobenius–Ruelle transfer operator
 $(R_W f)(x) = \sum_{\sigma(y)=x} W(y) f(y)$
xxxiv, 9, 11, 19, 26, 43, 45, 49,
51–57, 61, 64, 66, 76, 86, 91, 95,
100, 101, 105, 115, 116, 200
- \mathbb{R} : the real numbers
33, 10, 14, 195, 199
- \mathcal{R} : envelope of a fractal
195–199
- s : Hausdorff dimension
14, 17, 71, 72, 77
- $S := F^*$: adjoint operator
67
- S_i, S_i^*, T_i, T_i^* : the operators (isometries) and their adjoints (with stars) in a representation of the Cuntz relations (i.e., of the Cuntz algebra)
131, 132, 135, 161, 181, 182, 184,
201, 211, 213, 214, 219
- $\mathbb{T} := \{z \in \mathbb{C} \mid |z| = 1\}$: circle group, or one-torus
 $\cong \mathbb{R}/\mathbb{Z} \cong [0, 1)$
25, 32, 60, 61, 190, 204

U_2 : dyadic scaling operator 200	$Z_n(x, \omega)$: canonical martingale 50, 51
V : cocycle, i.e., a measurable function on $X \times \Omega$ such that $V(\tau_{\omega_1} x; (\omega_2, \omega_3, \dots)) = V(x; \omega)$ 43, 49, 92	\mathbb{Z} : the integers 5, 19, 22, 59, 66
V_0, V_1, V_n : resolution subspaces 22, 33, 104, 111, 123–128	$\mathbb{Z}_2 := \{0, 1\}$: cyclic group of order 2 27
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W : a measurable function $X \rightarrow [0, 1]$ xxxiv, 7–12, 17–21, 36, 41–45, 48, 49, 51–57, 61–66, 69, 71, 76, 77, 84–91, 101, 104, 105, 112–115, 117, 140, 141, 162	δ : Kronecker delta function 15, 46, 47, 104, 116, 131, 139, 163, 164, 183, 192, 211, 216, 219
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X, X_3, X_4, \bar{X}_4 : Cantor sets 14, 21, 71–80, 176	μ : the Haar measure, or other measure specified in the text 14, 41, 43, 61, 72, 77, 79, 136–139, 167, 168, 195, 198
$X_k(\omega) = \omega_k$: coordinate functions on a probability space 50	μ : multiplicity function 114, 117
(X, \mathcal{B}) : a set X with a σ -algebra \mathcal{B} of measurable subsets 6, 40, 84, 114, 115	$\mu \circ \sigma^{-1}$: is the measure given by $(\mu \circ \sigma^{-1})(B) := \mu(\sigma^{-1}(B))$ 52, 72
$z := e^{i2\pi t}$: Fourier variable 32	ν : Perron–Frobenius–Ruelle measure, or other measure specified in the text xxxiv, 52–54, 101, 105

- ρ : representation or state
47, 48, 139–141, 154
- σ : one-sided shift, an onto map
(actually endomorphism) $X \rightarrow X$
such that $\#\sigma^{-1}(\{x\})$ is constant
xxxiv, 6–8, 12, 17, 41, 45, 47,
51–54, 62, 64, 71, 74–76, 84,
89–91, 101, 114, 115, 159, 160,
170–172, 184, 186, 188
- σ^Ω : shift on Ω
47, 51, 52
- $\sigma^{-1}(B)$: pre-image under the mapping
 $\sigma := \{x \in X \mid \sigma(x) \in B\}$
6, 41, 72, 52, 84
- $(\sigma^\Omega)^{-1}$: pre-image under the mapping
 σ^Ω
52
- $\tau_0, \dots, \tau_{N-1}$: branches of σ^{-1} , maps
 $X \rightarrow X$ such that $\sigma \circ \tau_i = \text{id}_X$
7, 41, 47, 52, 72, 89, 115, 159
- τ_i^Ω : branches of $(\sigma^\Omega)^{-1}$
47, 48, 52
- φ : scaling function
3, 10, 12, 13, 15, 23, 102, 103, 114,
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- $\varphi_0, \varphi_1, \varphi_2, \dots$: wavelet packet system
112, 113, 118–122, 168, 191
- χ : characteristic function
14, 16, 47
- ψ : wavelet function
13, 16, 23, 102, 103, 134
- $\psi_{n,k}$: wavelets
15
- $\omega(k)$: representation in Ω of
 $k \in \mathbb{N}_0$: If $k =$
 $\omega_1 + \omega_2 N + \dots + \omega_n N^{n-1}$
is the Euclid N -adic representa-
tion, $\omega(k) :=$
 $(\omega_1, \dots, \omega_n, \underbrace{0, 0, 0, \dots}_{\infty \text{ string of zeroes}})$
11, 18, 77, 79, 92, 101, 122, 130,
135, 137, 138, 140
- Ω : probability space
 $:= \{0, 1, \dots, N-1\}^\mathbb{N}$
 $= \prod_{\mathbb{N}} \{0, 1, \dots, N-1\}$
 $=$ all functions:
 $\mathbb{N} \rightarrow \{0, 1, \dots, N-1\}$
 $= \{(\omega_1, \omega_2, \dots) \mid \omega_i \in \{0, 1, \dots, N-1\}\}$
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69, 85, 135
- $(\Omega, \mathcal{B}, \nu)$: probability space
56, 203
- $\mathbf{0}$: one-sided infinite string of zeroes
 $= (0, 0, 0, \dots) \in \Omega$
 ∞ string of zeroes
12, 85, 116, 130
- $\{\mathbf{0}\}$: the set with the one element $\mathbf{0}$
12, 85, 116
- $\mathbb{1}_{\mathcal{H}}$: identity operator (see
also I)
114, 115, 117, 122, 160, 161, 181,
182, 184
- $\mathbb{1}$: constant function equal to 1
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- $*$ -algebra, $*$ -isomorphism
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- $*$ -automorphism
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\vee : lattice operation applied to closed subspaces in a Hilbert space: the lim sup lattice operation 169, 181	\otimes : tensor product 139, 158, 161–163, 165, 170–172, 180–183, 189, 190, 193, 194, 197
\wedge : lattice operation applied to closed subspaces in a Hilbert space: the lim inf lattice operation 169, 181	\ominus : relative orthogonal complement 169
\emptyset : empty set 171, 172, 185,	\times : Cartesian product 11, 43, 49, 52, 88, 117, 130, 164, 166, 185, 188, 218
\bar{E} : closure of a set E 44, 172	$\#$: counting function 6, 41, 72, 101, 159, 184, 196
$\hat{\varphi}$: Fourier transform (of the scaling function φ) 10, 114, 111	$\langle \cdot \cdot \rangle$: inner product 16, 75, 77, 79, 104, 114, 140
\ll : relatively absolutely continuous (relation between measures) 50, 53	$ \cdot\rangle$: Dirac vector 160–163, 171, 182, 184, 185, 189, 193, 194
\uparrow : up-sampling 124, 132, 213, 214	$[\cdot, \cdot)$: interval closed to the left and open to the right 41, 61, 63, 65, 136–138, 165, 166, 167, 192
\downarrow : down-sampling 124–128, 132, 133, 212, 213, 215	$[\cdot, \cdot)$: segment of \mathbb{N}_0 165–167, 186, 188
\oplus : direct (orthogonal) sum 112, 172, 218	$[\cdot, \cdot]$: interval closed at both ends 7, 11, 13, 16–18, 47, 62–66, 71, 77, 84, 89–92, 102, 105, 112, 125, 130, 135–139, 195

Index

Comments on the use of the index: Some terms in the index may appear in the text in a slight variant, or variation of the actual index-term itself. For example, we will have terms in the index referring to “theorem so and so.” But when we use the Stone–Weierstraß *theorem*, I just say Stone–Weierstraß. The word “theorem” will be suppressed. It is implicitly understood.

Similarly, I often just say, “by domination” (or some variant thereof), when I mean, “by an application of the dominated convergence theorem,” or more fully: “By Lebesgue’s dominated convergence theorem.” It will be the same theorem whether the name is abbreviated or not.

For Fubini, the word “theorem” may be implicitly understood. Guido Weiss has made a verb out of it: “Fubinate” means “to exchange the order of two integrals.”

Similarly, the name Fatou often is used to mean “Fatou’s lemma” (the one about \liminf). For some reason poor Fatou only got credit for a lemma. But I do not mind upgrading him to a theorem, although “Fatou’s theorem” usually refers to the one about existence a.e. of boundary values of bounded harmonic functions. I usually call that one “the Fatou-Primalov theorem.”

- \mathcal{A} -random variable, *see* random variable, \mathcal{A} -
- abelian, *see* algebra, abelian; group, abelian;
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