Math 3900 Midterm
March 7, 2017

SHOW ALL WORK

[15] 1.) Create the single linkage dendrogram for the data set with distance matrix

$$\begin{array}{c} \\ b \\ c \\ d \\ e \end{array} \begin{array}{c} \begin{array}{ccccc} a & b & c & d & e \end{array} \\ \begin{pmatrix} 7 & & & & \\ 2 & 5 & & & \\ 10 & 3 & 8 & & \\ 6 & 1 & 4 & 4 & \end{pmatrix} \end{array}$$

List the clusters at each merging height:

The clusters at time 0 are $\{a\},\ \{b\},\ \{c\},\ \{d\},\ \{e\}$

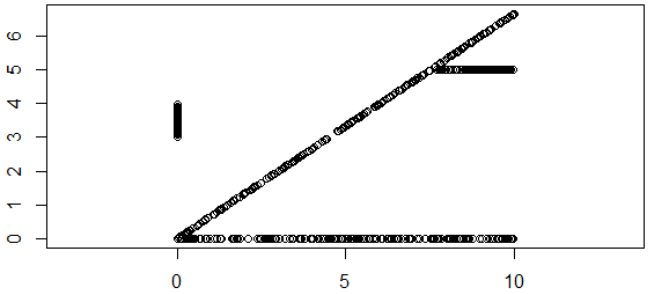
The clusters at merging height = _____ are _____

The clusters at merging height = _____ are _____

The clusters at merging height = _____ are _____

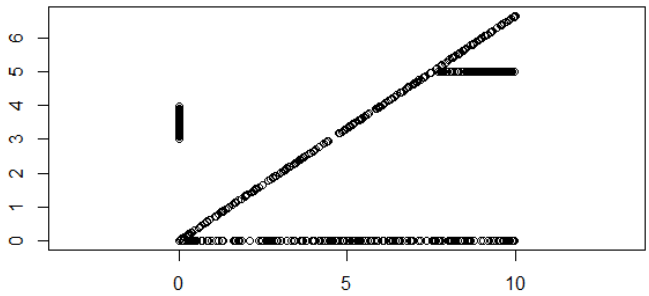
The clusters at merging height = _____ are _____

[20] 2.) Suppose TDA mapper is used to analyze the following data set where filter function = projection to the x-axis and 2 bins are created using 2 intervals with 50% overlap,

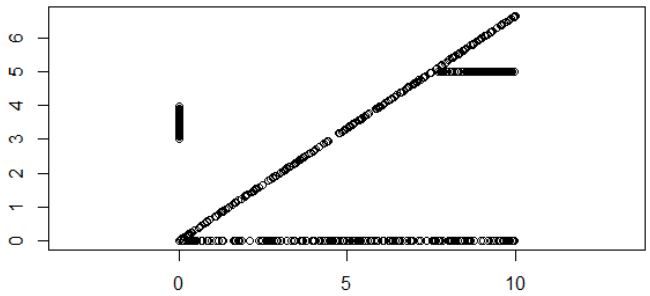


a.) Draw one rectangle in each of the figures below to indicate a bin:

Bin 1

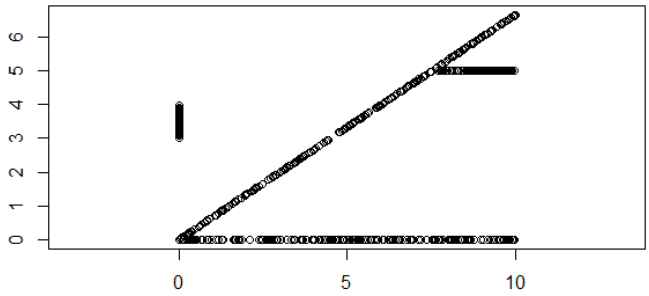


Bin 2

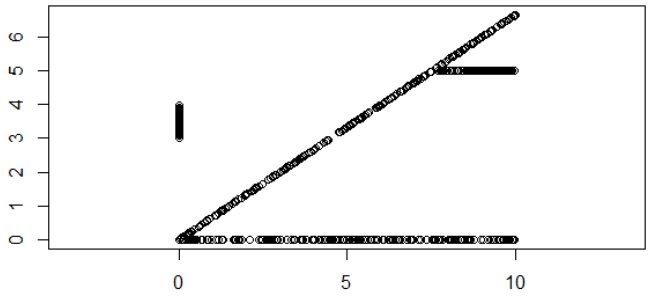


b.) Circle the clusters in Bin 1 in the figure below on the left, and circle the clusters in Bin 2 in the figure below on the right:

Bin 1



Bin 2



c.) Based on your answers above, draw the output of TDA mapper:

[6] 3.) For the following matrix, A, find its eigenvalues and corresponding eigenvectors. For each eigenvalue, find both its algebraic multiplicity and the geometric multiplicity.

$A = \begin{pmatrix} 1 & 2 \\ 1 & 2 \end{pmatrix}$

An eigenvalue of A is _____. It has algebraic multiplicity = _____ and geometric multiplicity = _____.

An eigenvector corresponding to this eigenvalue is _____.

A 2nd eigenvalue of A is _____ with algebraic multiplicity = _____ and geometric multiplicity = _____.

An eigenvector corresponding to this eigenvalue is _____.

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[4] 4.) Calculate the following, given the following distance matrix:

$$\begin{array}{c} \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{pmatrix} 7 & & & \\ 2 & 5 & & \\ 10 & 3 & 8 & \end{pmatrix} \end{array}$$

$\delta_1(b) =$ _____ $\delta_2(b) =$ _____ $\delta_3(b) =$ _____ $\delta_4(b) =$ _____

Recall $\delta_k(b)$ refers to the knn distance.

Multiple Choice: **Circle the best answer.**

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[5] 5.) The key idea(s) of topology that make extracting patterns via TDA mapper possible are

i.) Coordinate free: There is no dependence on the coordinate system chosen. Thus one can compare data derived from different platforms.

ii.) Invariant under small deformations and thus less sensitive to noise.

iii.) The output of TDA mapper gives compressed representations of the shape of the data (depending on filter function used).

iv.) All of the above.

v.) None of the above.

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[5] 6.) One **benefit** to the variety of parameters one can choose in TDA mapper is that

i.) You can try a variety of parameters until you get the output that you want.

ii.) You can probe the data set from a variety of different perpectives.

iii.) You can look for green jelly beans.

iv.) You may get a false positive.

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[5] 7.) If two vertices are close in the graph created by TDA mapper, then the data points represented by these vertices are close in the original data set.

i.) True ii.) False

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[5] 8.) An edge of the graph created by TDA mapper can represent the intersection of two clusters from the same bin.

i.) True ii.) False

~~[5] 9.) In the video *Applications of TDA to the Understanding of Disease and Drug Discovery*, Pek Lum, Ayasdi, in the diabetes graph, what does it mean when a node is colored red?~~

~~i.) This node represents a patient that must have diabetes.~~

~~ii.) This node represents a cluster of patients who all must have type II diabetes.~~

~~iii.) This node represents a cluster of patients where most of them have type II diabetes.~~

~~iv.) This node represents a cluster of patients where 50% have type II diabetes.~~

~~v.) None of the above.~~

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[5] 10.) The output of python mapper applied to a particular data set is given below. How many data points are in this data set?

i.) 1 ii.) 2 iii.) 3 iv.) 4 v.) 5

vi.) There is insufficient information to determine the number of data points in this data set.

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[5] 11.) Suppose you want to analyze flight departure delays at a small airport over a period of 365 days. Five airlines fly out of this airport. If 100 flights depart from this airport every day and if each coordinate of an element $\mathbf{x}$ in your data set represents the delay time in minutes for the ith departure of the day, then your data set lives in $\mathbb{R}^n$ where $n =$

i.) 1 ii.) 5 iii.) 12 iv.) 20 v.) 60 vi.) 100 vii.) 365

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[5] 12.) Suppose my data set consists of the following points:

$$(1, 3), (1.1, 3), (0.9, 3), (4, 3), (7, 3), (0.1, 7), (4, 7), (5, 7)$$

If my filter function is projection to the y-axis and I use 2 intervals with 50% overlap to determine my overlapping bins, then the output of TDA mapper will contain k edges, where $k =$

i.) 0 ii.) 1 iii.) 2 iv.) 3 v.) 4 vi.) 5 vii.) 6 viii.) 7 ix.) 8

x.) $0 \leq k \leq 8$ xi.) $1 \leq k \leq 8$ xii.) $0 \leq k \leq \frac{(8)(7)}{2}$ xiii.) $1 \leq k \leq \frac{(8)(7)}{2}$

~~[5] 13.) Suppose A is a **symmetric** matrix whose characteristic polynomial is $(r-1)^2(r+3)$. What can you say about the rank of A?~~

~~i.) 0, 1, 2~~ ~~ii.) 0, 1~~ ~~iii.) 0, 2~~ ~~iv.) 1, 2~~

~~v.) 1, 2, 3~~ ~~vi.) 1, 3~~ ~~vii.) 2, 3~~ ~~viii.) 1, 2~~

~~ix.) 0~~ ~~x.) 1~~ ~~xi.) 2~~ ~~xi.) 3~~

~~[5] 14.) Suppose M is a **symmetric** matrix whose characteristic polynomial is $r(r-4)^3(r+4)^2$. What can you say about the geometric multiplicity of the eigenvalue 4?~~

~~i.) 0, 1, 2~~ ~~ii.) 0, 1~~ ~~iii.) 0, 2~~ ~~iv.) 1, 2~~

~~v.) 1, 2, 3~~ ~~vi.) 1, 3~~ ~~vii.) 2, 3~~ ~~viii.) 1, 2~~

~~ix.) 0~~ ~~x.) 1~~ ~~xi.) 2~~ ~~xi.) 3~~

~~[5] 15.) Suppose A is a 9x9 matrix, and suppose 0 is an eigenvalue of A with algebraic multiplicity = geometric multiplicity = 4. Also assume that 3 is an eigenvalue of A with algebraic multiplicity = geometric multiplicity = 5. What is the rank of A?~~

~~i.) 1~~ ~~ii.) 2~~ ~~iii.) 3~~ ~~iv.) 4~~

~~v.) 5~~ ~~vi.) 6~~ ~~vii.) 7~~ ~~viii.) 8~~