Solving homogeneous equations: Ax = 0

Putting answer in parametric vector form

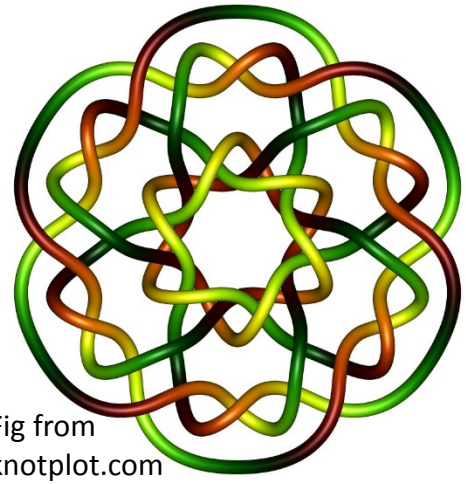

Fig from
knotplot.com

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University of Iowa

Solving homogeneous equations: Ax = 0

Putting answer in parametric vector form

or

Determining the solution set for Ax = 0

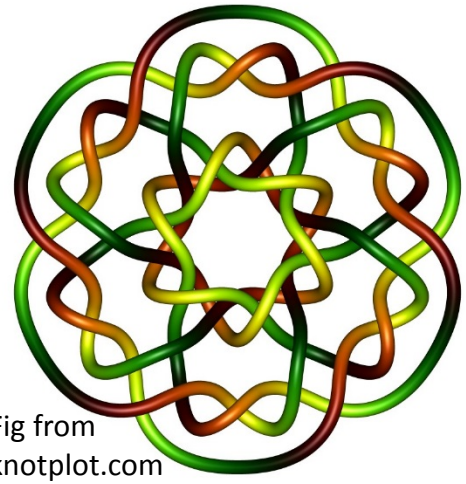

Fig from
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Solving homogeneous equations: Ax = 0

Putting answer in parametric vector form

or

Determining the solution set for Ax = 0

Nullspace of A = solution set for Ax = 0

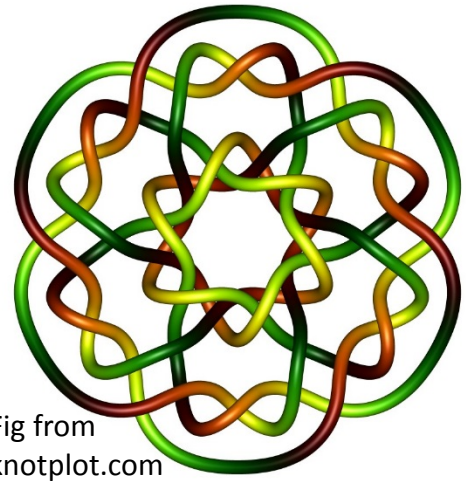

Fig from knotplot.com

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Solve: A **x** = **0** where A = $\begin{bmatrix} 1 & -10 & -24 & -42 \\ 1 & -8 & -18 & -32 \\ -2 & 20 & 51 & 87 \end{bmatrix}$

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Put A into echelon form and then into reduced echelon form:

$\begin{bmatrix} 1 & -10 & -24 & -42 \\ 1 & -8 & -18 & -32 \\ -2 & 20 & 51 & 87 \end{bmatrix}$ → $\begin{bmatrix} 1 & -10 & -24 & -42 \\ 0 & 2 & 6 & 10 \\ 0 & 0 & 3 & 3 \end{bmatrix}$

$R_2 - R_1 \rightarrow R_2$
$R_3 + 2R_1 \rightarrow R_3$

→ $\begin{bmatrix} 1 & -10 & 0 & -18 \\ 0 & 2 & 0 & 4 \\ 0 & 0 & 1 & 1 \end{bmatrix}$ → $\begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{bmatrix}$

$R_1 + 8R_3 \rightarrow R_1$
$R_1 - 2R_3 \rightarrow R_1$
$R_3/3 \rightarrow R_3$

$R_1 + 5R_2 \rightarrow R_1$
$R_2/2 \rightarrow R_2$

Solve: A **x** = **0** where A $\sim \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{bmatrix}$

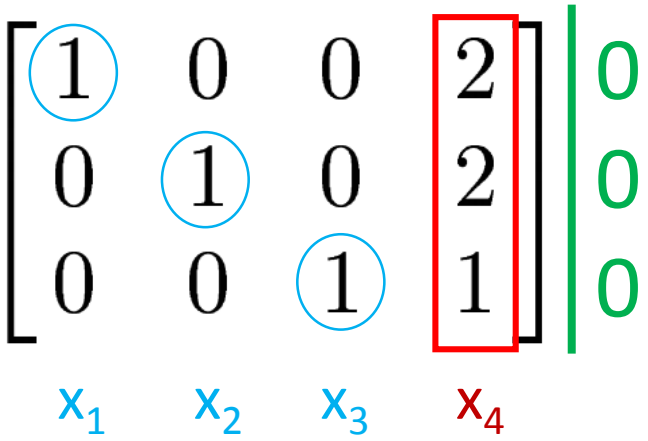
Put A into echelon form and then into reduced echelon form:

$$\begin{bmatrix} 1 & -10 & -24 & -42 \\ 1 & -8 & -18 & -32 \\ -2 & 20 & 51 & 87 \end{bmatrix} \xrightarrow[R_3 + 2R_1 \to R_3]{R_2 - R_1 \to R_2} \begin{bmatrix} 1 & -10 & -24 & -42 \\ 0 & 2 & 6 & 10 \\ 0 & 0 & 3 & 3 \end{bmatrix}$$

$$\xrightarrow[R_1 - 2R_3 \to R_1]{R_1 + 8R_3 \to R_1 \\ R_3/3 \to R_3} \begin{bmatrix} 1 & -10 & 0 & -18 \\ 0 & 2 & 0 & 4 \\ 0 & 0 & 1 & 1 \end{bmatrix} \xrightarrow[R_2/2 \to R_2]{R_1 + 5R_2 \to R_1} \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

Solve: A $\mathbf{x} = \mathbf{0}$ where A $\sim$ $\left[\begin{array}{cccc|c} 1 & 0 & 0 & 2 & 0 \\ 0 & 1 & 0 & 2 & 0 \\ 0 & 0 & 1 & 1 & 0 \end{array}\right]$

Solve: A **x** = **0** where A ~



Solve: $A\mathbf{x} = \mathbf{0}$ where $A \sim$

$$\left[\begin{array}{cccc|c} \textcircled{1} & 0 & 0 & 2 & 0 \\ 0 & \textcircled{1} & 0 & 2 & 0 \\ 0 & 0 & \textcircled{1} & 1 & 0 \end{array}\right]$$

x_1 x_2 x_3 x_4

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -2x_4 \\ -2x_4 \\ -x_4 \\ x_4 \end{bmatrix}$$

Solve: $A\mathbf{x} = \mathbf{0}$ where $A \sim$

$$\left[\begin{array}{cccc|c} \textcircled{1} & 0 & 0 & 2 & 0 \\ 0 & \textcircled{1} & 0 & 2 & 0 \\ 0 & 0 & \textcircled{1} & 1 & 0 \end{array}\right]$$

$x_1 \quad x_2 \quad x_3 \quad x_4$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -2x_4 \\ -2x_4 \\ -x_4 \\ x_4 \end{bmatrix} = \begin{bmatrix} -2 \\ -2 \\ -1 \\ 1 \end{bmatrix} x_4$$

Solve: B $\mathbf{x} = \mathbf{0}$ where B $\sim$ $\begin{bmatrix} 0 & 1 & 0 & 8 & 0 \\ 0 & 0 & 1 & -6 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

Solve: B **x** = **0** where B ~

$$\left[\begin{array}{ccccc|c} 0 & 1 & 0 & 8 & 0 & 0 \\ 0 & 0 & 1 & -6 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{array}\right]$$

x_1 x_2 x_3 x_4 x_5

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} =$$

Solve: B **x** = **0** where B ~

$$\left[\begin{array}{ccccc|c} 0 & 1 & 0 & 8 & 0 & 0 \\ 0 & 0 & 1 & -6 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{array}\right]$$

x_1 x_2 x_3 x_4 x_5

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} x_1 \\ \\ \\ x_4 \\ \end{bmatrix}$$

Solve: $B\mathbf{x} = \mathbf{0}$ where $B \sim$

$$\left[\begin{array}{ccccc|c} 0 & 1 & 0 & 8 & 0 & 0 \\ 0 & 0 & 1 & -6 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{array}\right]$$

x_1 x_2 x_3 x_4 x_5

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} x_1 \\ -8x_4 \\ -6x_4 \\ x_4 \\ 0 \end{bmatrix} = \begin{bmatrix} x_1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ -8x_4 \\ -6x_4 \\ x_4 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} x_1 + \begin{bmatrix} 0 \\ -8 \\ -6 \\ 1 \\ 0 \end{bmatrix} x_4$$

Solve: C **x** = **0** where C $\sim$ $\begin{bmatrix} 1 & 0 & 0 & 0 & 5 \\ 0 & 1 & 0 & 0 & -3 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

Solve: C **x** = **0** where C ~

$$\left[\begin{array}{ccccc|c} \textcircled{1} & 0 & 0 & 0 & 5 & 0 \\ 0 & \textcircled{1} & 0 & 0 & -3 & 0 \\ 0 & 0 & \textcircled{1} & 0 & 0 & 0 \\ 0 & 0 & 0 & \textcircled{1} & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

x_1 x_2 x_3 x_4 x_5

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} \quad \\ \quad \\ \quad \\ \quad \\ \quad \end{bmatrix}$$

Solve: $C\,\mathbf{x} = \mathbf{0}$ where $C \sim$

$$\left[\begin{array}{ccccc|c} \textcircled{1} & 0 & 0 & 0 & 5 & 0 \\ 0 & \textcircled{1} & 0 & 0 & -3 & 0 \\ 0 & 0 & \textcircled{1} & 0 & 0 & 0 \\ 0 & 0 & 0 & \textcircled{1} & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

x_1 x_2 x_3 x_4 x_5

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} -5x_5 \\ 3x_5 \\ 0 \\ x_5 \\ x_5 \end{bmatrix} = \begin{bmatrix} -5 \\ 3 \\ 0 \\ 1 \\ 1 \end{bmatrix} x_5$$

Solve: D **x** = **0** where D $\sim \begin{bmatrix} 1 & 0 & 0 & -7 & 2 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 5 & -4 & 0 \end{bmatrix}$

Solve: D **x** = **0** where D ~

$$\left[\begin{array}{cccccc|c} \textcircled{1} & 0 & 0 & -7 & 2 & 0 & 0 \\ 0 & \textcircled{1} & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & \textcircled{1} & 5 & -4 & 0 & 0 \end{array}\right]$$

x_1 x_2 x_3 x_4 x_5 x_6

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} \quad \\ \quad \\ \quad \\ \quad \\ \quad \\ \quad \end{bmatrix}$$

Solve: D **x** = **0** where D ~

$$\left[\begin{array}{cccccc|c} \textcircled{1} & 0 & 0 & -7 & 2 & 0 & 0 \\ 0 & \textcircled{1} & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & \textcircled{1} & 5 & -4 & 0 & 0 \end{array}\right]$$

x_1 x_2 x_3 x_4 x_5 x_6

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} \\ x_4 \\ x_5 \\ x_6 \\ \\ \end{bmatrix}$$

Solve: D **x** = **0** where D ~

$$\left[\begin{array}{cccccc|c} \textcircled{1} & 0 & 0 & -7 & 2 & 0 & 0 \\ 0 & \textcircled{1} & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & \textcircled{1} & 5 & -4 & 0 & 0 \end{array}\right]$$

x_1 x_2 x_3 x_4 x_5 x_6

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} 7x_4 - 2x_5 \\ -x_4 \\ -5x_4 + 4x_5 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} 7 \\ -1 \\ -5 \\ 1 \\ 0 \\ 0 \end{bmatrix} x_4 + \begin{bmatrix} -2 \\ 0 \\ 4 \\ 0 \\ 1 \\ 0 \end{bmatrix} x_5 + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} x_6$$

Solve: E **x** = **0** where E $\sim$ $\begin{bmatrix} 0 & 1 & 0 & -5 & 0 & 0 & 5 \\ 0 & 0 & 1 & 7 & 0 & 0 & -3 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

Solve: E **x** = **0** where E ~

$$\begin{bmatrix} 0 & 1 & 0 & -5 & 0 & 0 & 5 \\ 0 & 0 & 1 & 7 & 0 & 0 & -3 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

x_1 x_2 x_3 x_4 x_5 x_6 x_7

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{bmatrix} =$$

Solve: E **x** = **0** where E ~

$$\begin{bmatrix} 0 & 1 & 0 & -5 & 0 & 0 & 5 \\ 0 & 0 & 1 & 7 & 0 & 0 & -3 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

x_1 x_2 x_3 x_4 x_5 x_6 x_7

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{bmatrix} = \begin{bmatrix} x_1 \\ 5x_4 - 5x_7 \\ -7x_4 + 3x_7 \\ x_4 \\ 0 \\ x_7 \\ x_7 \end{bmatrix} = \begin{bmatrix} \\ \\ \\ \\ \\ \\ \end{bmatrix} x_1 + \begin{bmatrix} \\ \\ \\ \\ \\ \\ \end{bmatrix} x_4 + \begin{bmatrix} \\ \\ \\ \\ \\ \\ \end{bmatrix} x_7$$

Solve: E **x** = **0** where E ~

$$\begin{bmatrix} 0 & 1 & 0 & -5 & 0 & 0 & 5 \\ 0 & 0 & 1 & 7 & 0 & 0 & -3 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

x_1 x_2 x_3 x_4 x_5 x_6 x_7

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{bmatrix} = \begin{bmatrix} x_1 \\ 5x_4 - 5x_7 \\ -7x_4 + 3x_7 \\ x_4 \\ 0 \\ x_7 \\ x_7 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} x_1 + \begin{bmatrix} 0 \\ 5 \\ -7 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} x_4 + \begin{bmatrix} 0 \\ -5 \\ 3 \\ 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} x_7$$