1. Let $A = \begin{bmatrix} 1 & 3 & 10 & 6 \\ -3 & 7 & 0 & 6 \\ -2 & 2 & -5 & 0 \end{bmatrix}$

[8] **1a.)** Find a basis for the column space of A:________________________________

[2] **1b.)** Rank(A) = _______

[2] **1c.)** Nullity(A) = _______

[3] **1d.)** Are columns of A linearly independent?_______

[5] **1e.)** If possible write one of the columns of A as a linear combination of the other columns of A.

1. Let $A = \begin{bmatrix} 1 & 4 & 3 & 5 \\ -2 & 2 & -5 & 0 \\ -5 & 0 & -13 & -5 \end{bmatrix}$

[8] **1a.)** Find a basis for the column space of A:________________________________

[2] **1b.)** Rank(A) = _______

[2] **1c.)** Nullity(A) = _______

[3] **1d.)** Are columns of A linearly independent?_______

[5] **1e.)** If possible write one of the columns of A as a linear combination of the other columns of A.

1. Let $A = \begin{bmatrix} 1 & -4 & 5 & 3 \\ -2 & 2 & -5 & 0 \\ -3 & 0 & -5 & 3 \end{bmatrix}$

[8] **1a.)** Find a basis for the column space of A:______________________________

[2] **1b.)** Rank(A) = _______

[2] **1c.)** Nullity(A) = _______

[3] **1d.)** Are columns of A linearly independent?_______

[5] **1e.)** If possible write one of the columns of A as a linear combination of the other columns of A.

1. Let $A = \begin{bmatrix} 1 & 3 & 4 & 5 \\ -2 & 10 & 2 & 0 \\ -5 & 17 & 0 & -5 \end{bmatrix}$

[8] **1a.)** Find a basis for the column space of A:________________

[2] **1b.)** Rank(A) = ________

[2] **1c.)** Nullity(A) = ________

[3] **1d.)** Are columns of A linearly independent?________

[5] **1e.)** If possible write one of the columns of A as a linear combination of the other columns of A.

3. Let $A = \begin{bmatrix} 12 & 4 & 0 & 0 \\ -30 & -10 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}$.

[12] **3a).** Find an invertible matrix P and a diagonal matrix D such that $D = P^{-1}AP$.

Note, you may use the following facts:

(1.) A has eigenvalue $\lambda_1 = 0$ with multiplicity 1.

(2.) A has eigenvalue λ_2 with multiplicity 3.

(3.) The vector $\begin{bmatrix} 0 \\ 0 \\ -3 \\ 1 \end{bmatrix}$ is an eigenvector of A.

$P =$ ________________, $D =$ ________________

[3] **3b.** The characteristic polynomial of the matrix $A =$ ________________

3. Let $A = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 12 & 30 \\ 0 & 0 & -4 & -10 \end{bmatrix}$.

[12] **3a).** Find an invertible matrix P and a diagonal matrix D such that $D = P^{-1}AP$.

Note, you may use the following facts:

(1.) A has eigenvalue $\lambda_1 = 0$ with multiplicity 1.

(2.) A has eigenvalue λ_2 with multiplicity 3.

(3.) The vector $\begin{bmatrix} -2 \\ 5 \\ 0 \\ 0 \end{bmatrix}$ is an eigenvector of A.

$P =$ ______________________________, $D =$ ______________________________

[3] **3b.** The characteristic polynomial of the matrix $A =$ ______________________________

3. Let $A = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 12 & -30 \\ 0 & 0 & 4 & -10 \end{bmatrix}$.

[12] **3a).** Find an invertible matrix P and a diagonal matrix D such that $D = P^{-1}AP$.

Note, you may use the following facts:

(1.) A has eigenvalue $\lambda_1 = 0$ with multiplicity 1.

(2.) A has eigenvalue λ_2 with multiplicity 3.

(3.) The vector $\begin{bmatrix} 2 \\ 5 \\ 0 \\ 0 \end{bmatrix}$ is an eigenvector of A.

$P =$ ____________________, $D =$ ____________________

[3] **3b.** The characteristic polynomial of the matrix $A =$ ____________________

3. Let $A = \begin{bmatrix} 12 & -4 & 0 & 0 \\ 30 & -10 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}$.

[12] **3a).** Find an invertible matrix P and a diagonal matrix D such that $D = P^{-1}AP$.

Note, you may use the following facts:

(1.) A has eigenvalue $\lambda_1 = 0$ with multiplicity 1.

(2.) A has eigenvalue λ_2 with multiplicity 3.

(3.) The vector $\begin{bmatrix} 0 \\ 0 \\ 3 \\ 1 \end{bmatrix}$ is an eigenvector of A.

$P =$ ________________________________, $D =$ ________________________________

[3] **3b.** The characteristic polynomial of the matrix $A =$ ________________________________

[3] **4a.** Calculate the dot product: $\begin{bmatrix} 1 \\ 3 \\ 3 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ -1 \\ -1 \\ 3 \end{bmatrix} =$ ______

[8] **4b.** Find the orthogonal projection of $v = \begin{bmatrix} 5 \\ -4 \\ 1 \\ 0 \end{bmatrix}$ onto the subspace W of $\mathbb{R}^3$ spanned by $\begin{bmatrix} 1 \\ 3 \\ 3 \\ 1 \end{bmatrix}$ & $\begin{bmatrix} 3 \\ -1 \\ -1 \\ 3 \end{bmatrix}$

$\text{proj}_W(v) =$ ______________________________

[6] **4c.** Use the Gram-Schmidt process to determine an orthonormal basis for the subspace of $\mathbb{R}^4$ spanned by $\begin{bmatrix} 1 \\ 3 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ -1 \\ 3 \end{bmatrix}, \begin{bmatrix} 5 \\ -4 \\ 1 \\ 0 \end{bmatrix}$

Basis = ______________________________

[3] **4a.** Calculate the dot product: $\begin{bmatrix} 1 \\ 3 \\ 3 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ -1 \\ -1 \\ 3 \end{bmatrix} =$ ______

[8] **4b.** Find the orthogonal projection of $v = \begin{bmatrix} 5 \\ 1 \\ -4 \\ 0 \end{bmatrix}$ onto the subspace W of $\mathbb{R}^3$ spanned by $\begin{bmatrix} 1 \\ 3 \\ 3 \\ 1 \end{bmatrix}$ & $\begin{bmatrix} 3 \\ -1 \\ -1 \\ 3 \end{bmatrix}$

$\text{proj}_W(v) =$ ______________________________

[6] **4c.** Use the Gram-Schmidt process to determine an orthonormal basis for the subspace of $\mathbb{R}^4$ spanned by $\begin{bmatrix} 1 \\ 3 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ -1 \\ 3 \end{bmatrix}, \begin{bmatrix} 5 \\ 1 \\ -4 \\ 0 \end{bmatrix}$

Basis = ______________________________

[2] **5.** Circle the correct answer:

Suppose $A\vec{x} = \vec{b}$ has a unique solution, then $A\vec{x} = \vec{0}$ has

- B. Unique solution

---

6. Fill in the SIX blanks below:

Suppose that A is a 7×9 matrix which has a 3 pivot columns, then

[2] **6a.** The rank of A = 3

[2] **6b.** The nullity of A = 6

[4] **6c.** The column space of A is a 3 dimensional subspace of R^k where k =7

[4] **6d.** The nullspace of A is a 6 dimensional subspace of R^n where n =9