Math 2550 Matrix Algebra Exam #2 (Form A)
April 9, 2014 SHOW ALL WORK and SIMPLIFY ANSWERS

[15] 1.) Use Cramer's rule to solve the following system of equations for y
Note: Only solve for y. You do not need to solve for x or z

$$\begin{array}{rcrcrcl} & & 2y & + & 5z & = & 0 \\ 3x & + & y & - & 4z & = & 0 \\ 2x & + & 3y & + & z & = & 10 \end{array}$$

Answer: $y =$ ________________

2.) Let $D = \begin{bmatrix} e^{5t} & 2e^{-4t} \\ 5e^{5t} & -8e^{-4t} \end{bmatrix}$.

[10] 2a.) $detD =$ ____

[10] 2b.) Does the matrix D have an inverse? ________. If so, find D^{-1}

Answer: $D^{-1} =$ __________________________

Extra credit:

[1] 2c.) Are the columns of D linearly independent? ________

[1] 2d.) Are the rows of D linearly independent? ________

[1] 2e.) Solve $D\mathbf{x} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

Answer: $\mathbf{x} =$ __________________________

3.) Let $E = \begin{bmatrix} 1 & 5 \\ 5 & 1 \end{bmatrix}$

[4] 3a.) The characteristic polynomial of the matrix E is ____________________

[16] 3b.) Find the eigenvalues of E and a basis for each eigenspace.

$\lambda_1 =$ _______ is an eigenvalue. A basis for its eigenspace is

$\lambda_2 =$ _______ is an eigenvalue. A basis for its eigenspace is

4.) Suppose $A = \begin{bmatrix} 1 & 2 & -4 & 3 & 2 \\ -5 & 2 & 20 & 6 & 11 \\ 2 & 2 & -8 & 9 & 7 \end{bmatrix}$

Suppose also that A is row equivalent to $C = \begin{bmatrix} 1 & 0 & -4 & 6 & 5 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & 3 \end{bmatrix}$

Hint: Recall definition of reduced echelon form.

[10] 4a.) The nullspace of $A =$ ______________________________

[4] 4b.) A basis for the nullspace of $A =$ ______________________________

[10] 4c.) The column space of A= ______________________________

[2] 4d.) Do the columns of A span R^3? ________

[2] 4e.) Do the columns of A span R^5? ________

5.) Suppose A is a 4×8 matrix with rank 2,

[3] 5a.) The nullity of A is ________

[3] 5b.) The dimension of the column space of $A =$ ________

[2] 5c.) The nullspace of A is a subspace of R^a where $a =$ ________

[2] 5d.) The column space of A is a subspace of R^b where $b =$ ________

6.) Let A be a square matrix. Circle T for true and F for False.

[2] 6a.) $det A = 0$ if and only if $A\mathbf{x} = \mathbf{0}$ has an infinite number of solutions.
T F

[2] 6b.) 0 is an eigenvalue of A if and only if $A\mathbf{x} = \mathbf{0}$ has an infinite number of solutions. T F

[3] 6c.) If $\mathbf{v}$ is an eigenvector of A corresponding to eigenvalue λ_0, then $2\mathbf{v}$ is also an eigenvector of A corresponding to eigenvalue λ_0 T F