Math 2550 Matrix Algebra Exam #2 (Form A)
April 9, 2014 SHOW ALL WORK and SIMPLIFY ANSWERS

[15] 1.) Use Cramer's rule to solve the following system of equations for y
Note: Only solve for y. You do not need to solve for x or z

$$\begin{array}{ccccccc} & & 5y & + & 2z & = & 0 \\ 3x & + & y & - & 4z & = & 0 \\ 2x & + & 3y & + & z & = & 10 \end{array}$$

$$\begin{vmatrix} 0 & 5 & 2 \\ 3 & 1 & -4 \\ 2 & 3 & 1 \end{vmatrix} = -5(3+8) + 2(9-2) = -55 + 14 = -41$$

or equivalently:

$$\begin{vmatrix} 0 & 5 & 2 \\ 3 & 1 & -4 \\ 2 & 3 & 1 \end{vmatrix} = -3(5-6) + 2(-20-2) = 3 - 44 = -41$$

replacing 2nd column:

$$\begin{vmatrix} 0 & 0 & 2 \\ 3 & 0 & -4 \\ 2 & 10 & 1 \end{vmatrix} = 2(30-0) = -10(0-6) = 60$$

Answer: $y = -\frac{60}{41}$

2.) Let $D = \begin{bmatrix} e^{4t} & 2e^{-3t} \\ 4e^{4t} & -6e^{-3t} \end{bmatrix}$.

$$\begin{vmatrix} e^{4t} & 2e^{-3t} \\ 4e^{4t} & -6e^{-3t} \end{vmatrix} = e^{4t}(-6e^{-3t}) - 4e^{4t}(2e^{-3t}) = -6e^t - 8e^t = -14e^t$$

$$D^{-1} = \frac{1}{14e^t} \begin{bmatrix} -6e^{-3t} & -2e^{-3t} \\ -4e^{4t} & e^{4t} \end{bmatrix} = \begin{bmatrix} \frac{3e^{-4t}}{7} & \frac{e^{-4t}}{7} \\ \frac{2e^{3t}}{7} & -\frac{e^{3t}}{14} \end{bmatrix}.$$

[10] 2a.) $detD = \underline{-14e^t}$

[10] 2b.) Does the matrix D have an inverse? *yes*. If so, find D^{-1}

Answer: $D^{-1} = \begin{bmatrix} \frac{3e^{-4t}}{7} & \frac{e^{-4t}}{7} \\ \frac{2e^{3t}}{7} & -\frac{e^{3t}}{14} \end{bmatrix}$

Extra credit:

[1] 2c.) Are the columns of D linearly independent? *yes*

[1] 2d.) Are the rows of D linearly independent? *yes*

[1] 2e.) Solve $D\mathbf{x} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

$$D\mathbf{x} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$D^{-1}D\mathbf{x} = D^{-1}\begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\mathbf{x} = \begin{bmatrix} \frac{3e^{-4t}}{7} & \frac{e^{-4t}}{7} \\ \frac{2e^{3t}}{7} & -\frac{e^{3t}}{14} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{e^{-4t}}{7} \\ -\frac{e^{3t}}{14} \end{bmatrix}$$

Answer: $\mathbf{x} = \begin{bmatrix} \frac{e^{-4t}}{7} \\ -\frac{e^{3t}}{14} \end{bmatrix}$

3.) Let $E = \begin{bmatrix} 1 & 6 \\ 6 & 1 \end{bmatrix}$

$$\begin{vmatrix} 1-\lambda & 6 \\ 6 & 1-\lambda \end{vmatrix} = (1-\lambda)^2 - 36 = \lambda^2 - 2\lambda - 35 = (\lambda+5)(\lambda-7) = 0$$

$\lambda = -5$: $\begin{bmatrix} 6 & 6 \\ 6 & 6 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$

Thus $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -x_2 \\ x_2 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix} x_2$

$\lambda = 7$: $\begin{bmatrix} -6 & 6 \\ 6 & -6 \end{bmatrix} \sim \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}$

Thus $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_2 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} x_2$

[4] 3a.) The characteristic polynomial of the matrix E is $\underline{(\lambda+5)(\lambda-7)}$

[16] 3b.) Find the eigenvalues of E and a basis for each eigenspace.

$\lambda_1 = \underline{-5}$ is an eigenvalue. A basis for its eigenspace is $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$

$\lambda_2 = \underline{7}$ is an eigenvalue. A basis for its eigenspace is $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

4.) Suppose $A = \begin{bmatrix} 1 & 2 & -4 & 3 & 5 \\ -7 & 2 & 28 & 6 & -8 \\ 2 & 2 & -8 & 9 & 13 \end{bmatrix}$

Suppose also that A is row equivalent to $C = \begin{bmatrix} 1 & 0 & -4 & 6 & 8 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & 3 \end{bmatrix}$

Hint: Recall definition of reduced echelon form.

$$C \sim \begin{bmatrix} 1 & 0 & -4 & 0 & 2 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

Thus if $A\mathbf{x} = \mathbf{0}$, then $C\mathbf{x} = \mathbf{0}$ and $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 4x_3 - 2x_5 \\ 0 \\ x_3 \\ -x_5 \\ x_5 \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} x_3 + \begin{bmatrix} -2 \\ 0 \\ 0 \\ -1 \\ 1 \end{bmatrix} x_5$

[10] 4a.) The nullspace of $A =$ $span\{ \begin{bmatrix} 4 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 0 \\ 0 \\ -1 \\ 1 \end{bmatrix} \}$

[4] 4b.) A basis for the nullspace of $A =$ $\{ \begin{bmatrix} 4 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 0 \\ 0 \\ -1 \\ 1 \end{bmatrix} \}$

[10] 4c.) The column space of $A=$ $span \left\{ \begin{bmatrix} 1 \\ -7 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 6 \\ 9 \end{bmatrix} \right\}$

[2] 4d.) Do the columns of A span R^3? *yes*

[2] 4e.) Do the columns of A span R^5? *no*

5.) Suppose A is a 3×7 matrix with rank 2,

[3] 5a.) The nullity of A is $\underline{5}$

[3] 5b.) The dimension of the column space of $A = \underline{2}$

[2] 5c.) The nullspace of A is a subspace of R^a where $a = \underline{7}$

[2] 5d.) The column space of A is a subspace of R^b where $b = \underline{3}$

6.) Let A be a square matrix. Circle T for true and F for False.

[2] 6a.) $detA = 0$ if and only if $A\mathbf{x} = \mathbf{0}$ has an infinite number of solutions.

T

[2] 6b.) 0 is an eigenvalue of A if and only if $A\mathbf{x} = \mathbf{0}$ has an infinite number of solutions.

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[3] 6c.) If $\mathbf{v}$ is an eigenvector of A corresponding to eigenvalue λ_0, then $2\mathbf{v}$ is also an eigenvector of A corresponding to eigenvalue λ_0

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