Math 2550 Matrix Algebra Exam #1 (Form D)
Feb. 26, 2014 SHOW ALL WORK

[30] 1.) Solve the following systems of equations. Write your answer in parametric vector format (note this is a multipart question).

1a.)
$$\begin{bmatrix} 2 & -4 & -36 & 26 & -12 \\ 7 & -1 & 4 & 0 & 2 \\ 0 & 0 & 0 & 0 & 3 \\ 1 & 0 & 2 & -1 & 1 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Answer:____

1b.)

$$\begin{bmatrix} 2 & -4 & -36 & 26 & -12 \\ 7 & -1 & 4 & 0 & 2 \\ 0 & 0 & 0 & 0 & 3 \\ 1 & 0 & 2 & -1 & 1 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 4 \\ 6 \\ 0 \\ 1 \end{bmatrix}$$

Answer:____

1c.)

$$\begin{bmatrix} 2 & -4 & -36 & 26 & -12 \\ 7 & -1 & 4 & 0 & 2 \\ 0 & 0 & 0 & 0 & 3 \\ 1 & 0 & 2 & -1 & 1 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 30 \\ 0 \\ 15 \\ 0 \end{bmatrix}$$

Answer:____

[20] 2.) Let A be the coefficient matrix from problem 1.

2a.) Is $\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ in the span of the columns of A?_______________

If so, write $\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ as a linear combination of the columns of A:

2b.) Is $\begin{bmatrix} 4 \\ 6 \\ 0 \\ 1 \end{bmatrix}$ in the span of the columns of A?_______________

If so, write $\begin{bmatrix} 4 \\ 6 \\ 0 \\ 1 \end{bmatrix}$ as a linear combination of the columns of A:

2c.) Is $\begin{bmatrix} 30 \\ 0 \\ 15 \\ 0 \end{bmatrix}$ in the span of the columns of A?_______________

If so, write $\begin{bmatrix} 30 \\ 0 \\ 15 \\ 0 \end{bmatrix}$ as a linear combination of the columns of A:

3.) Let A be the coefficient matrix from problem 1.

[4] 3a.) Are the columns of A linearly independent? _______________

[4] 3b.) Do the columns of A span R^4? _______________

[1 point extra credit] 3c.) Given an example of 4 vectors that span R^4 where 3 of your vectors are columns of A. Briefly explain.

[10] 4a.) Find the inverse of $\begin{bmatrix} 3 & 12 \\ 2 & 10 \end{bmatrix}$

Answer:____

[4] 4b.) Use the inverse found in part a to solve the following system of equations:

$$3x + 12\ y = 0$$
$$2x + 10y = 1$$

Answer:____

[5] 5.) Given an example of matrices A and B where neither are the zero matrix, but $AB = 0$.

Answer: $A =$ ________________________ $, B =$ ________________________

6.) Circle T for true and F for False (watch out for trick(s)).

[3] 6a.) If A, B, C are matrices and $AB = AC$, then $B = C$ T F

[3] 6b.) If A, B, C are square matrices and $AB = AC$, then $B = C$ T F

[4] 6c.) If A and B are matrices, then $AB = BA$ T F

[4] 6.) If A is a 3×3 matrix and the equation $A\mathbf{x} = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$ had a unique solution, then A is invertible. T F

Circle the correct answer:

[5] 7a.) Suppose $A\mathbf{x} = \mathbf{0}$ has a unique solution, then given a vector $\mathbf{b}$ of the appropriate dimension, $A\mathbf{x} = \mathbf{b}$ has

A. No solution

B. Unique solution

C. Infinitely many solutions

D. at most one solution

E. either no solution or an infinite number of solutions

F. either a unique solution or an infinite number of solutions

G. no solution, a unique solution or an infinite number of solutions, depending on the system of equations

H. none of the above

[5] 7b.) Suppose A is a SQUARE matrix and $A\mathbf{x} = \mathbf{0}$ has a unique solution, then given a vector $\mathbf{b}$ of the appropriate dimension, $A\mathbf{x} = \mathbf{b}$ has

A. No solution

B. Unique solution

C. Infinitely many solutions

D. at most one solution

E. either no solution or an infinite number of solutions

F. either a unique solution or an infinite number of solutions

G. no solution, a unique solution or an infinite number of solutions, depending on the system of equations

H. none of the above