Note $\begin{bmatrix} \frac{1}{\sqrt{14}} & -\frac{3}{\sqrt{10}} \\ \frac{2}{\sqrt{14}} & 0 \\ \frac{3}{\sqrt{14}} & \frac{1}{\sqrt{10}} \end{bmatrix}$ is NOT invertible since it is not square, but

since the columns of $A = \begin{bmatrix} \frac{1}{\sqrt{14}} & -\frac{3}{\sqrt{10}} \\ \frac{2}{\sqrt{14}} & 0 \\ \frac{3}{\sqrt{14}} & \frac{1}{\sqrt{10}} \end{bmatrix} = [\,\mathbf{a_1} \quad \mathbf{a_2}\,]$ are orthonormal:

$$A^T A = \begin{bmatrix} \mathbf{a_1} \\ \mathbf{a_2} \end{bmatrix} [\,\mathbf{a_1} \quad \mathbf{a_2}\,] = \begin{bmatrix} \mathbf{a_1} \cdot \mathbf{a_1} & \mathbf{a_1} \cdot \mathbf{a_2} \\ \mathbf{a_2} \cdot \mathbf{a_1} & \mathbf{a_2} \cdot \mathbf{a_2} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\text{That is } \begin{bmatrix} \frac{1}{\sqrt{14}} & \frac{2}{\sqrt{14}} & \frac{3}{\sqrt{14}} \\ -\frac{3}{\sqrt{10}} & 0 & \frac{1}{\sqrt{10}} \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{14}} & -\frac{3}{\sqrt{10}} \\ \frac{2}{\sqrt{14}} & 0 \\ \frac{3}{\sqrt{14}} & \frac{1}{\sqrt{10}} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

BUT $\begin{bmatrix} \frac{1}{\sqrt{14}} & -\frac{3}{\sqrt{10}} \\ \frac{2}{\sqrt{14}} & 0 \\ \frac{3}{\sqrt{14}} & \frac{1}{\sqrt{10}} \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{14}} & \frac{2}{\sqrt{14}} & \frac{3}{\sqrt{14}} \\ -\frac{3}{\sqrt{10}} & 0 & \frac{1}{\sqrt{10}} \end{bmatrix} \neq I_3$

Solve $\begin{bmatrix} \frac{1}{\sqrt{14}} & -\frac{3}{\sqrt{10}} \\ \frac{2}{\sqrt{14}} & 0 \\ \frac{3}{\sqrt{14}} & \frac{1}{\sqrt{10}} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$ $\qquad [\mathbf{a_1} \quad \mathbf{a_2}]\,\mathbf{x} = \mathbf{b}$

Note the columns are orthoNORMAL. Thus

$$\begin{bmatrix} \frac{1}{\sqrt{14}} & \frac{2}{\sqrt{14}} & \frac{3}{\sqrt{14}} \\ -\frac{3}{\sqrt{10}} & 0 & \frac{1}{\sqrt{10}} \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{14}} & -\frac{3}{\sqrt{10}} \\ \frac{2}{\sqrt{14}} & 0 \\ \frac{3}{\sqrt{14}} & \frac{1}{\sqrt{10}} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{14}} & \frac{2}{\sqrt{14}} & \frac{3}{\sqrt{14}} \\ -\frac{3}{\sqrt{10}} & 0 & \frac{1}{\sqrt{10}} \end{bmatrix} \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} (\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}) \cdot (4,5,6) \\ (-\frac{3}{\sqrt{10}}, 0, \frac{1}{\sqrt{10}}) \cdot (4,5,6) \end{bmatrix} = \begin{bmatrix} \frac{4}{\sqrt{14}} + \frac{10}{\sqrt{14}} + \frac{18}{\sqrt{14}} \\ -\frac{3}{\sqrt{10}} + 0 + \frac{6}{\sqrt{10}} \end{bmatrix} = \begin{bmatrix} \frac{32}{\sqrt{14}} \\ \frac{3}{\sqrt{10}} \end{bmatrix}$$

You MUST check answer:

$$\begin{bmatrix} \frac{1}{\sqrt{14}} & -\frac{3}{\sqrt{10}} \\ \frac{2}{\sqrt{14}} & 0 \\ \frac{3}{\sqrt{14}} & \frac{1}{\sqrt{10}} \end{bmatrix} \begin{bmatrix} \frac{32}{\sqrt{14}} \\ \frac{3}{\sqrt{10}} \end{bmatrix} = \begin{bmatrix} \frac{32}{14} - \frac{9}{10} \\ \frac{32}{7} \\ \frac{96}{14} + \frac{3}{10} \end{bmatrix} = \begin{bmatrix} \frac{160-63}{70} \\ \frac{64}{14} \\ \frac{480+21}{14} \end{bmatrix} = \begin{bmatrix} \frac{97}{70} \\ \frac{32}{7} \\ \frac{501}{14} \end{bmatrix} \neq \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$$

Thus $\begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$ is NOT in $W = span \left\{ \begin{bmatrix} \frac{1}{\sqrt{14}} \\ \frac{2}{\sqrt{14}} \\ \frac{3}{\sqrt{14}} \end{bmatrix}, \begin{bmatrix} -\frac{3}{\sqrt{10}} \\ 0 \\ \frac{1}{\sqrt{10}} \end{bmatrix} \right\}$

But $proj_W \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} = \begin{bmatrix} \frac{97}{70} \\ \frac{32}{7} \\ \frac{501}{14} \end{bmatrix}$

Solve $\begin{bmatrix} 1 & -3 \\ 2 & 0 \\ 3 & 1 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$ $\qquad [\,\mathbf{c_1} \quad \mathbf{c_2}\,]\mathbf{x} = \mathbf{b}$

Note the columns are orthoGONAL. Thus

$$\begin{bmatrix} 1 & 2 & 3 \\ -3 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -3 \\ 2 & 0 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ -3 & 0 & 1 \end{bmatrix} \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$$

$$\begin{bmatrix} 14 & 0 \\ 0 & 10 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} (1,2,3) \cdot (4,5,6) \\ (-3,0,1) \cdot (4,5,6) \end{bmatrix} = \begin{bmatrix} 4+10+18 \\ -3+0+6 \end{bmatrix} = \begin{bmatrix} 32 \\ 3 \end{bmatrix}$$

Thus $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \frac{32}{14} \\ \frac{3}{10} \end{bmatrix}$

You MUST check answer:

$$\begin{bmatrix} 1 & -3 \\ 2 & 0 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} \frac{32}{14} \\ \frac{3}{10} \end{bmatrix} = \begin{bmatrix} \frac{32}{14} - \frac{9}{10} \\ \frac{32}{7} \\ \frac{96}{14} + \frac{3}{10} \end{bmatrix} = \begin{bmatrix} \frac{160-63}{70} \\ \frac{64}{14} \\ \frac{480+21}{14} \end{bmatrix} = \begin{bmatrix} \frac{97}{70} \\ \frac{32}{7} \\ \frac{501}{14} \end{bmatrix} \neq \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$$

Thus $\begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$ is NOT in $W = span \left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix} \right\}$

But $proj_W \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} = \begin{bmatrix} \frac{97}{70} \\ \frac{32}{7} \\ \frac{501}{14} \end{bmatrix} = (\frac{\mathbf{c_1} \cdot \mathbf{b}}{\mathbf{c_1} \cdot \mathbf{c_1}})\mathbf{c_1} + (\frac{\mathbf{c_2} \cdot \mathbf{b}}{\mathbf{c_2} \cdot \mathbf{c_2}})\mathbf{c_2}$

Solve $\begin{bmatrix} 1 & -3 \\ 2 & 0 \\ 3 & 1 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 6 \\ 10 \end{bmatrix}$ $\qquad [\mathbf{c_1} \quad \mathbf{c_2}]\mathbf{x} = \mathbf{d}$

Note the columns are orthoGONAL. Thus

$$\begin{bmatrix} 1 & 2 & 3 \\ -3 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -3 \\ 2 & 0 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ -3 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 6 \\ 10 \end{bmatrix}$$

$$\begin{bmatrix} 14 & 0 \\ 0 & 10 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} (1,2,3) \cdot (0,6,10) \\ (-3,0,1) \cdot (0,6,10) \end{bmatrix} = \begin{bmatrix} 0 + 12 + 30 \\ 0 + 0 + 10 \end{bmatrix} = \begin{bmatrix} 42 \\ 10 \end{bmatrix}$$

Thus $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \frac{42}{14} \\ \frac{10}{10} \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{\mathbf{c_1} \cdot \mathbf{d}}{\mathbf{c_1} \cdot \mathbf{c_1}} \\ \frac{\mathbf{c_2} \cdot \mathbf{d}}{\mathbf{c_2} \cdot \mathbf{c_2}} \end{bmatrix}$

You MUST check answer:

$$\begin{bmatrix} 1 & -3 \\ 2 & 0 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 - 3 \\ 6 + 0 \\ 9 + 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 6 \\ 10 \end{bmatrix}$$

Thus $\begin{bmatrix} 0 \\ 6 \\ 10 \end{bmatrix}$ is in $W = span \left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix} \right\} = span\{\mathbf{c_1}, \mathbf{c_2}\}$

AND $proj_{span\{\mathbf{c_1},\mathbf{c_2}\}}\mathbf{d} = proj_W \begin{bmatrix} 0 \\ 6 \\ 10 \end{bmatrix} = \begin{bmatrix} 0 \\ 6 \\ 10 \end{bmatrix} = (\frac{\mathbf{c_1} \cdot \mathbf{d}}{\mathbf{c_1} \cdot \mathbf{c_1}})\mathbf{c_1} + (\frac{\mathbf{c_2} \cdot \mathbf{d}}{\mathbf{c_2} \cdot \mathbf{c_2}})\mathbf{c_2}$