

1. (1 pt) local/Library/UI/LinearSystems/mformsHint.pg

Determine the following equivalent representations of the following system of equations:

$$3x + 8y = -7$$

$$-8x + 7y = -38$$

a. Find the augmented matrix of the system.

$$\left[\begin{array}{cc|c} _ & _ & _ \\ _ & _ & _ \end{array} \right]$$

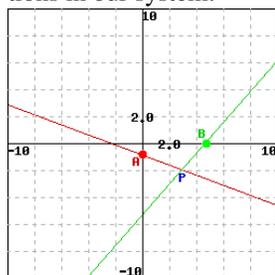
b. Find the matrix form of the system.

$$\left[\begin{array}{cc} _ & _ \\ _ & _ \end{array} \right] \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} _ \\ _ \end{bmatrix}$$

c. Find matrices that satisfy the following matrix equation.

$$x \begin{bmatrix} _ \\ _ \end{bmatrix} + y \begin{bmatrix} _ \\ _ \end{bmatrix} = \begin{bmatrix} _ \\ _ \end{bmatrix}$$

d. The graph below shows the lines determined by the two equations in our system:



Find the coordinates of

$$P = (_, _)$$

Find the coordinates of y-intercept of the red line.

$$A = (0, _)$$

Find the coordinates of x-intercept of the green line.

$$B = (_, 0)$$

Hint: (Instructor hint preview: show the student hint after 1 attempts. The current number of attempts is 0.)

For part d, note that you are given the equations of the red and green lines. If a point lies on a line, it must satisfy the equation for that line.

2. (1 pt) Library/Rochester/setLinearAlgebra9Dependence-ur.la.9.10.pg

Express the vector $v = \begin{bmatrix} 26 \\ 13 \end{bmatrix}$ as a linear combination of

$$x = \begin{bmatrix} 6 \\ 5 \end{bmatrix} \text{ and } y = \begin{bmatrix} 1 \\ 3 \end{bmatrix}.$$

$$v = _ x + _ y.$$

3. (1 pt) local/Library/UI/LinearSystems/linearCombHW4.pg

$$\text{Let } A = \begin{bmatrix} 1 & -1 & -3 \\ 4 & -2 & -10 \\ -2 & 1 & 5 \end{bmatrix} \text{ and } b = \begin{bmatrix} -1 \\ -11 \\ 5 \end{bmatrix}$$

? 1. Determine if b is a linear combination of a_1 , a_2 and a_3 , the columns of the matrix A .

If it is a linear combination, determine a non-trivial linear relation - (a non-trivial relation is three numbers which are not all three zero.) Otherwise, enter 0's for the coefficients.

$$_ a_1 + _ a_2 + _ a_3 = b.$$

4. (1 pt) Library/WHFreeman/Holt_linear_algebra/Chaps.1-4/2.2.8.pg

$$\text{Let } \mathbf{a}_1 = \begin{bmatrix} -6 \\ -1 \end{bmatrix} \text{ and } \mathbf{b} = \begin{bmatrix} -24 \\ -4 \end{bmatrix}.$$

Is \mathbf{b} in the span of \mathbf{a}_1 ?

- A. No, \mathbf{b} is not in the span.
- B. Yes, \mathbf{b} is in the span.
- C. We cannot tell if \mathbf{b} is in the span.

Either fill in the coefficients of the vector equation, or enter "NONE" if no solution is possible.

$$\mathbf{b} = _ \mathbf{a}_1$$

5. (1 pt) Library/WHFreeman/Holt_linear_algebra/Chaps.1-4/2.2.10.pg

$$\text{Let } \mathbf{a}_1 = \begin{bmatrix} -6 \\ 4 \\ 7 \end{bmatrix}, \mathbf{a}_2 = \begin{bmatrix} -7 \\ 2 \\ 9 \end{bmatrix}, \text{ and } \mathbf{b} = \begin{bmatrix} -31 \\ 18 \\ 37 \end{bmatrix}.$$

Is \mathbf{b} in the span of \mathbf{a}_1 and \mathbf{a}_2 ?

- A. Yes, \mathbf{b} is in the span.
- B. No, \mathbf{b} is not in the span.
- C. We cannot tell if \mathbf{b} is in the span.

Either fill in the coefficients of the vector equation, or enter "NONE" if no solution is possible.

$$\mathbf{b} = _ \mathbf{a}_1 + _ \mathbf{a}_2$$

6. (1 pt) local/Library/UI/LinearSystems/spanHW4.pg

$$\text{Let } A = \begin{bmatrix} 1 & -2 & 5 \\ 5 & -11 & 27 \\ 1 & -5 & 12 \end{bmatrix}, \text{ and } b = \begin{bmatrix} -4 \\ 2 \\ -4 \end{bmatrix}.$$

Denote the columns of A by a_1 , a_2 , a_3 , and let $W = \text{span}\{a_1, a_2, a_3\}$.

? 1. Determine if b is in $\{a_1, a_2, a_3\}$

? 2. Determine if b is in W

How many vectors are in $\{a_1, a_2, a_3\}$? (For infinitely many, enter -1) _____

How many vectors are in W ? (For infinitely many, enter -1) _____

7. (1 pt) Library/TCNJ/TCNJ_VectorEquations/problem4.pg

Let $u = \begin{bmatrix} -1 \\ 5 \\ -3 \end{bmatrix}$ and $v = \begin{bmatrix} 1 \\ -7 \\ 4 \end{bmatrix}$

Find two vectors in $\text{span}\{u, v\}$ that are not multiples of u or v and show the weights on u and v used to generate them.

____ u + ____ v = _____

____ u + ____ v = _____

8. (1 pt) Library/TCNJ/TCNJ_VectorEquations/problem6.pg

Let $\vec{u} = \begin{bmatrix} 5 \\ -1 \\ 1 \end{bmatrix}$ and $\vec{v} = \begin{bmatrix} -10 \\ 1 \\ 0 \end{bmatrix}$. Find a vector \vec{w} **not** in $\text{span}\{\vec{u}, \vec{v}\}$.

$\vec{w} = \begin{bmatrix} \text{---} \\ \text{---} \\ \text{---} \end{bmatrix}$

9. (1 pt) Library/TCNJ/TCNJ_MatrixEquations/problem4.pg

Let $A = \begin{bmatrix} -1 & 1 & 5 \\ -2 & 3 & 3 \\ -2 & -3 & 1 \end{bmatrix}$ and $x = \begin{bmatrix} 2 \\ -2 \\ 4 \end{bmatrix}$.

1. What does Ax mean?

10. (1 pt) Library/Rochester/setLinearAlgebra9Dependence/ur_la_9.8.pg

Let $v_1 = \begin{bmatrix} 2 \\ 4 \\ -1 \end{bmatrix}$, $v_2 = \begin{bmatrix} -2 \\ -6 \\ -1 \end{bmatrix}$, and $y = \begin{bmatrix} 2 \\ 10 \\ h \end{bmatrix}$.

For what value of h is y in the plane spanned by v_1 and v_2 ?
 $h =$ _____

11. (1 pt) Library/WHFreeman/Holt_linear_algebra/Chaps.1-4/2.2.31.pg

Let $A = \begin{bmatrix} 3 & 3 \\ 5 & 4 \\ -3 & 8 \end{bmatrix}$.

We want to determine if the system $Ax = \mathbf{b}$ has a solution for every $\mathbf{b} \in \mathbb{R}^3$.

Select the best answer.

- A. There is a solution for every $\mathbf{b} \in \mathbb{R}^3$ but we need to row reduce A to show this.
- B. There is a solution for every $\mathbf{b} \in \mathbb{R}^3$ since $2 < 3$
- C. There is a not solution for every $\mathbf{b} \in \mathbb{R}^3$ but we need to row reduce A to show this.
- D. There is not a solution for every $\mathbf{b} \in \mathbb{R}^3$ since $2 < 3$.

- E. We cannot tell if there is a solution for every $\mathbf{b} \in \mathbb{R}^3$.

12. (1 pt) Library/TCNJ/TCNJ_VectorEquations/problem5.pg

Let $H = \text{span}\{u, v\}$. For each of the following sets of vectors determine whether H is a line or a plane.

1. $u = \begin{bmatrix} -8 \\ -3 \\ -1 \end{bmatrix}$, $v = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$,

2. $u = \begin{bmatrix} 4 \\ 4 \\ 4 \end{bmatrix}$, $v = \begin{bmatrix} -16 \\ -15 \\ -13 \end{bmatrix}$,

3. $u = \begin{bmatrix} 2 \\ -1 \\ -3 \end{bmatrix}$, $v = \begin{bmatrix} -8 \\ 4 \\ 12 \end{bmatrix}$,

4. $u = \begin{bmatrix} 5 \\ 1 \\ -2 \end{bmatrix}$, $v = \begin{bmatrix} 13 \\ 2 \\ -2 \end{bmatrix}$,

13. (1 pt) Library/TCNJ/TCNJ_MatrixEquations/problem13.pg

Do the following sets of vectors span \mathbb{R}^3 ?

1. $\begin{bmatrix} -1 \\ 2 \\ -2 \end{bmatrix}$, $\begin{bmatrix} -3 \\ 6 \\ -7 \end{bmatrix}$, $\begin{bmatrix} -7 \\ 14 \\ -16 \end{bmatrix}$, $\begin{bmatrix} -11 \\ 22 \\ -26 \end{bmatrix}$

2. $\begin{bmatrix} 3 \\ -2 \\ 3 \end{bmatrix}$, $\begin{bmatrix} -3 \\ 2 \\ -4 \end{bmatrix}$, $\begin{bmatrix} -3 \\ 2 \\ -5 \end{bmatrix}$

3. $\begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}$, $\begin{bmatrix} -3 \\ -3 \\ 7 \end{bmatrix}$

4. $\begin{bmatrix} 1 \\ -3 \\ 3 \end{bmatrix}$, $\begin{bmatrix} -7 \\ 5 \\ 7 \end{bmatrix}$, $\begin{bmatrix} -12 \\ 7 \\ 17 \end{bmatrix}$

14. (1 pt) Library/TCNJ/TCNJ_MatrixEquations/problem1.pg

Show that the vectors

$\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 1 \\ 4 \\ 1 \end{bmatrix}$

do not span \mathbb{R}^3 by giving a vector not in their span.

$\begin{bmatrix} \text{---} \\ \text{---} \\ \text{---} \end{bmatrix}$

If a linear system has four equations and seven variables, then it must have infinitely many solutions.

- A. True
- B. False

Every linear system with free variables has infinitely many solutions.

- A. True
- B. False

Any linear system with more variables than equations cannot have a unique solution.

- A. True
- B. False

If a linear system has the same number of equations and variables, then it must have a unique solution.

- A. True
- B. False

A vector b is a linear combination of the columns of a matrix A if and only if the equation $Ax = b$ has at least one solution.

- A. True
- B. False

If the columns of an $m \times n$ matrix, A span \mathbb{R}^m , then the equation, $Ax = b$ is consistent for each b in \mathbb{R}^m .

- A. True
- B. False

If A is an $m \times n$ matrix and if the equation $Ax = b$ is inconsistent for some b in \mathbb{R}^m , then A cannot have a pivot position in every row.

- A. True
- B. False

Any linear combination of vectors can always be written in the form Ax for a suitable matrix A and vector x .

- A. True
- B. False

If the equation $Ax = b$ is inconsistent, then b is not in the set spanned by the columns of A .

- A. True
- B. False

If A is an $m \times n$ matrix whose columns do not span \mathbb{R}^m , then the equation $Ax = b$ is inconsistent for some b in \mathbb{R}^m .

- A. True
- B. False

The equation $Ax = b$ is consistent if the augmented matrix $[A \ b]$ has a pivot position in every row.

- A. True
- B. False

If the augmented matrix $[A \ b]$ has a pivot position in every row, then the equation $Ax = b$ is inconsistent.

- A. True
- B. False

16. (1 pt) Library/WHFreeman/Holt_linear_algebra/Chaps.1-4/2.2.56.pg

What conditions on a matrix A insures that $Ax = \mathbf{b}$ has a solution for all \mathbf{b} in \mathbb{R}^n ?

Select the best statement. (The best condition should work with any positive integer n .)

- A. The equation will have a solution for all \mathbf{b} in \mathbb{R}^n as long as the columns of A do not include the zero column.
- B. The equation will have a solution for all \mathbf{b} in \mathbb{R}^n as long as the columns of A span \mathbb{R}^n .
- C. There is no easy test to determine if the equation will have a solution for all \mathbf{b} in \mathbb{R}^n .
- D. The equation will have a solution for all \mathbf{b} in \mathbb{R}^n as long as no column of A is a scalar multiple of another column.

- E. none of the above

17. (1 pt) Library/WHFreeman/Holt_linear_algebra/Chaps.1-4/2.2.57.pg

Assume $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ spans \mathbb{R}^3 .
Select the best statement.

- A. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ spans \mathbb{R}^3 unless \mathbf{u}_4 is the zero vector.
- B. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ always spans \mathbb{R}^3 .
- C. There is no easy way to determine if $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ spans \mathbb{R}^3 .
- D. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ never spans \mathbb{R}^3 .
- E. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ spans \mathbb{R}^3 unless \mathbf{u}_4 is a scalar multiple of another vector in the set.
- F. none of the above

18. (1 pt) Library/WHFreeman/Holt_linear_algebra/Chaps.1-4/2.2.58.pg

Assume $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ does not span \mathbb{R}^3 .
Select the best statement.

- A. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ may, but does not have to, span \mathbb{R}^3 .
- B. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ never spans \mathbb{R}^3 .
- C. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ spans \mathbb{R}^3 unless \mathbf{u}_4 is a scalar multiple of another vector in the set.
- D. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ spans \mathbb{R}^3 unless \mathbf{u}_4 is the zero vector.
- E. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ always spans \mathbb{R}^3 .
- F. none of the above

19. (1 pt) Library/TCNJ/TCNJ_SolutionSetsLinearSystems/problem8.pg

Suppose the solution set of a certain system of equations can be described as $x_1 = 4 - 6t$, $x_2 = 4 + 6t$, $x_3 = 4t - 3$, $x_4 = -6 - 4t$, where t is a free variable. Use vectors to describe this solution set as a line in \mathbb{R}^4 .

$$L(t) = \begin{bmatrix} _ \\ _ \\ _ \\ _ \end{bmatrix} + t \begin{bmatrix} _ \\ _ \\ _ \\ _ \end{bmatrix}.$$

20. (1 pt) Library/NAU/setLinearAlgebra/HomLinEq.pg
Solve the equation

$$-6x - 2y + 9z = 0$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = s \begin{bmatrix} _ \\ _ \\ _ \end{bmatrix} + t \begin{bmatrix} _ \\ _ \\ _ \end{bmatrix}$$

21. (1 pt) local/Library/UI/LinearSystems/ur_la.1.19AxB.pg

Solve the system

$$\begin{cases} 4x_1 - 3x_2 + 4x_3 + 4x_4 = 0 \\ -x_1 + x_2 + 3x_3 + 3x_4 = 0 \\ 3x_1 - 2x_2 + 7x_3 + 7x_4 = 0 \\ 3x_1 - 3x_2 - 9x_3 - 9x_4 = 0 \end{cases}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = + \begin{bmatrix} _ \\ _ \\ _ \\ _ \end{bmatrix} + s \begin{bmatrix} _ \\ _ \\ _ \\ _ \end{bmatrix} + t \begin{bmatrix} _ \\ _ \\ _ \\ _ \end{bmatrix}.$$

Solve the system

$$\begin{cases} 4x_1 - 3x_2 + 4x_3 + 4x_4 = 3 \\ -x_1 + x_2 + 3x_3 + 3x_4 = 4 \\ 3x_1 - 2x_2 + 7x_3 + 7x_4 = 7 \\ 3x_1 - 3x_2 - 9x_3 - 9x_4 = -12 \end{cases}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} _ \\ _ \\ _ \\ _ \end{bmatrix} + \begin{bmatrix} _ \\ _ \\ _ \\ _ \end{bmatrix} + s \begin{bmatrix} _ \\ _ \\ _ \\ _ \end{bmatrix} + t \begin{bmatrix} _ \\ _ \\ _ \\ _ \end{bmatrix}.$$

If the matrix A corresponds to the coefficient matrix for the above system of equations, then given any vector \vec{b} , the matrix equation $A\vec{x} = \vec{b}$ will always have an infinite number of solutions.

- A. True
- B. False

22. (1 pt) local/Library/UI/LinearSystems/ur_la.1.20vv3.pg

Solve the system

$$\begin{cases} x_1 + 4x_2 + 2x_3 + 2x_5 - 2x_6 = 0 \\ - x_4 + 4x_5 - 4x_6 = 0 \\ x_1 + 4x_2 + + 6x_5 - 4x_6 = 0 \end{cases}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} _ \\ _ \\ _ \\ _ \\ _ \\ _ \end{bmatrix} + s \begin{bmatrix} _ \\ _ \\ _ \\ _ \\ _ \\ _ \end{bmatrix} + t \begin{bmatrix} _ \\ _ \\ _ \\ _ \\ _ \\ _ \end{bmatrix} + u \begin{bmatrix} _ \\ _ \\ _ \\ _ \\ _ \\ _ \end{bmatrix}.$$

Solve the system

$$\begin{cases} x_1 + 4x_2 + 2x_3 + 2x_5 - 2x_6 = 1 \\ -x_4 + 4x_5 - 4x_6 = 7 \\ x_1 + 4x_2 + 6x_5 - 4x_6 = 5 \end{cases}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} - \\ - \\ - \\ - \\ - \\ - \end{bmatrix} + \begin{bmatrix} - \\ - \\ - \\ - \\ - \\ - \end{bmatrix} s + \begin{bmatrix} - \\ - \\ - \\ - \\ - \\ - \end{bmatrix} t + \begin{bmatrix} - \\ - \\ - \\ - \\ - \\ - \end{bmatrix} u.$$

If the matrix A corresponds to the coefficient matrix for the above system of equations, then given any vector \vec{b} , the matrix equation $A\vec{x} = \vec{b}$ will always have an infinite number of solutions.

- A. True
- B. False