

Calculate the determinant of  $\begin{bmatrix} -4.6666666666667 & 2 \\ -6 & 3 \end{bmatrix}$ .

- A. -4
- B. -3
- C. -2
- D. -1
- E. 0
- F. 1
- G. 2
- H. 3
- I. 4
- J. 5

*Correct Answers:*

- C

**2. (1 pt) local/Library/UI/Fall14/HW8.2.pg**

Evaluate the following  $3 \times 3$  determinant. Use the properties of determinants to your advantage.

$$\begin{vmatrix} -2 & 0 & -4 \\ -6 & 0 & 6 \\ -9 & 0 & 4 \end{vmatrix}$$

- A. -4
- B. -3
- C. -2
- D. -1
- E. 0
- F. 1
- G. 2
- H. 3
- I. 4
- J. none of the above

Does the matrix have an inverse?

- A. No
- B. Yes

*Correct Answers:*

- E
- A

**3. (1 pt) local/Library/UI/Fall14/HW8.3.pg**

Given the matrix

$$\begin{bmatrix} -5 & 0 & 2 \\ 0 & -1 & -5 \\ 0 & 0 & 5 \end{bmatrix}$$

(a) find its determinant

- A. 25
- B. -5
- C. -4
- D. -2
- E. -1
- F. 0
- G. 1
- H. 3
- I. 5
- J. 7
- K. None of those above

(b) does the matrix have an inverse?

- A. No
- B. Yes

*Correct Answers:*

- A
- B

**4. (1 pt) local/Library/UI/Fall14/HW8.4.pg**

If  $A$  and  $B$  are  $4 \times 4$  matrices,  $\det(A) = 4$ ,  $\det(B) = -3$ , then  $\det(AB) =$

- A. -15
- B. -12
- C. -11
- D. -8
- E. -5
- F. 0
- G. 3
- H. 6
- I. 8
- J. 12
- K. None of those above

$$\det(2A) =$$

- A. -40
- B. 64
- C. -28
- D. -21
- E. -10
- F. -1
- G. 10
- H. 21
- I. 28
- J. 36
- K. 40
- L. None of those above

$$\det(A^T) =$$

- A. -3
- B. -2
- C. -1
- D. 0
- E. 1
- F. 2
- G. 3
- H. 4
- I. None of those above

$$\det(B^{-1}) =$$

- A. -0.5
- B. -0.4
- C. -0.333333333333333
- D. 0
- E. -0.333333333333333
- F. 0.4
- G. 0.5
- H. 1
- I. None of those above

$$\det(B^4) =$$

- A. -81
- B. -36
- C. -12
- D. 0
- E. 12
- F. 36
- G. 81
- H. 1024
- I. None of those above

*Correct Answers:*

- B
- B
- H
- E
- G

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**5. (1 pt) local/Library/UI/Fall14/HW8\_5.pg**

Find the determinant of the matrix

$$A = \begin{bmatrix} -1 & 0 & 0 & 0 \\ -3 & 2 & 0 & 0 \\ 8 & -4 & 4 & 0 \\ -7 & -2 & -8 & -5 \end{bmatrix}.$$

$$\det(A) =$$

- A. -400
- B. -360
- C. -288
- D. -120
- E. 0
- F. 120
- G. 40
- H. 240
- I. 360
- J. 400
- K. None of those above

*Correct Answers:*

- G

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**6. (1 pt) local/Library/UI/problem7.pg**

$A$  and  $B$  are  $n \times n$  matrices.

Adding a multiple of one row to another does not affect the determinant of a matrix.

- A. True
- B. False

If the columns of  $A$  are linearly dependent, then  $\det A = 0$ .

- A. True
- B. False

$$\det(A+B) = \det A + \det B.$$

- A. True
- B. False

*Correct Answers:*

- A
- A
- B

**7. (1 pt) local/Library/UI/Fall14/HW8.7.pg**

Suppose that a  $4 \times 4$  matrix  $A$  with rows  $v_1, v_2, v_3$ , and  $v_4$  has determinant  $\det A = -4$ . Find the following determinants:

$$B = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ 2v_4 \end{bmatrix} \det(B) =$$

- A. -18
- B. -15
- C. -12
- D. -8
- E. -9
- F. 0
- G. 9
- H. 12
- I. 15
- J. 18
- K. None of those above

$$C = \begin{bmatrix} v_4 \\ v_1 \\ v_2 \\ v_3 \end{bmatrix} \det(C) =$$

- A. -18
- B. 4
- C. -9
- D. -3
- E. 0
- F. 3
- G. 9
- H. 12
- I. 18
- J. None of those above

$$D = \begin{bmatrix} v_1 \\ v_2 \\ v_3 + 6v_4 \\ v_4 \end{bmatrix}$$

$$\det(D) =$$

- A. -18

- B. -4
- C. -9
- D. -3
- E. 0
- F. 3
- G. 9
- H. 12
- I. 18
- J. None of those above

*Correct Answers:*

- D
- B
- B

**8. (1 pt) local/Library/UI/Fall14/HW8.8.pg**

Use determinants to determine whether each of the following sets of vectors is linearly dependent or independent.

$$\begin{bmatrix} 7 \\ -4 \end{bmatrix}, \begin{bmatrix} -1 \\ -9 \end{bmatrix},$$

- A. Linearly Dependent
- B. Linearly Independent

$$\begin{bmatrix} 1 \\ 5 \\ -1 \end{bmatrix}, \begin{bmatrix} -2 \\ -13 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 3 \\ 3 \end{bmatrix},$$

- A. Linearly Dependent
- B. Linearly Independent

$$\begin{bmatrix} -3 \\ -9 \end{bmatrix}, \begin{bmatrix} -3 \\ -9 \end{bmatrix},$$

- A. Linearly Dependent
- B. Linearly Independent

$$\begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix}, \begin{bmatrix} -2 \\ -6 \\ -8 \end{bmatrix}, \begin{bmatrix} 3 \\ 9 \\ 12 \end{bmatrix},$$

- A. Linearly Dependent
- B. Linearly Independent

*Correct Answers:*

- B
- B
- A
- A

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**9. (1 pt) local/Library/UI/Fall14/HW8\_10.pg**

$$A = \begin{bmatrix} 3 & 3 & 0 & -9 \\ 5 & -9 & 0 & 0 \\ 0 & -9 & 0 & 0 \\ -2 & 1 & -3 & -9 \end{bmatrix}$$

The determinant of the matrix is

- A. -1890
- B. -1024
- C. -630
- D. 1215
- E. -210
- F. 0
- G. 324
- H. 630
- I. 1024
- J. None of those above

**Hint:** Find a good row or column and expand by minors.

*Correct Answers:*

- D

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**10. (1 pt) local/Library/UI/Fall14/HW8\_11.pg**

Find the determinant of the matrix

$$M = \begin{bmatrix} 2 & 0 & 0 & 2 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 & -1 \\ 0 & 0 & 0 & 2 & -2 \\ 0 & 3 & 1 & 0 & 0 \end{bmatrix}.$$

$$\det(M) =$$

- A. -48
- B. -35
- C. -20
- D. -5
- E. 5
- F. 18
- G. 20
- H. 81
- I. None of those above

*Correct Answers:*

- C

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**11. (1 pt) local/Library/UI/Fall14/HW8\_12.pg**

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \end{bmatrix}$$

And now for the grand finale: The determinant of the matrix is

- A. -362880
- B. -5
- C. 0
- D. 5
- E. 20
- F. 30
- G. 40
- H. 362880
- I. None of the above

**Hint:** Remember that a square linear system has a unique solution if the determinant of the coefficient matrix is non-zero.

**Solution:** (Instructor solution preview: show the student solution after due date. )

**Solution:** Since all the rows are the same a linear system with  $A$  as its coefficient matrix cannot have a unique solution and therefore the determinant of  $A$  is zero.

*Correct Answers:*

- C

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**12. (1 pt) local/Library/UI/4.3.1a.pg**

Find bases for the column space and the null space of matrix  $A$ . You should verify that the Rank-Nullity Theorem holds. An equivalent echelon form of matrix  $A$  is given to make your work easier.

$$A = \begin{bmatrix} 1 & 2 & 9 \\ 3 & 3 & 18 \\ 2 & 6 & 24 \end{bmatrix} \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\text{Basis for the column space of } A = \begin{bmatrix} \underline{\quad} \\ \underline{\quad} \\ \underline{\quad} \end{bmatrix} \begin{bmatrix} \underline{\quad} \\ \underline{\quad} \\ \underline{\quad} \end{bmatrix}$$

$$\text{Basis for the null space of } A = \begin{bmatrix} \underline{\quad} \\ \underline{\quad} \\ \underline{\quad} \end{bmatrix}$$

**Solution:** (Instructor solution preview: show the student solution after due date. )

**SOLUTION:**

A basis for the column space, determined from the pivot columns 1 and 2, is

$$\left\{ \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ 6 \end{bmatrix} \right\}$$

Solve  $Ax = 0$ , to obtain  $x = s \begin{bmatrix} -3 \\ -3 \\ 1 \end{bmatrix}$ , and so the nullspace basis is  $\left\{ \begin{bmatrix} -3 \\ -3 \\ 1 \end{bmatrix} \right\}$ .

*Correct Answers:*

- $\begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}$
- $\begin{bmatrix} 2 \\ 3 \\ 6 \end{bmatrix}$
- $\begin{bmatrix} -3 \\ -3 \\ 1 \end{bmatrix}$

**13. (1 pt) local/Library/UI/4.3.3.pg**

Find bases for the column space and the null space of matrix A. You should verify that the Rank-Nullity Theorem holds. An equivalent echelon form of matrix A is given to make your work easier.

$$A = \begin{bmatrix} 1 & 0 & -4 & -3 \\ -2 & 1 & 13 & 5 \\ 0 & 1 & 5 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -4 & -3 \\ 0 & 1 & 5 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\text{Basis for the column space of } A = \begin{bmatrix} \underline{\quad} \\ \underline{\quad} \\ \underline{\quad} \end{bmatrix} \begin{bmatrix} \underline{\quad} \\ \underline{\quad} \\ \underline{\quad} \end{bmatrix}$$

$$\text{Basis for the null space of } A = \begin{bmatrix} \underline{\quad} \\ \underline{\quad} \\ \underline{\quad} \end{bmatrix} \begin{bmatrix} \underline{\quad} \\ \underline{\quad} \\ \underline{\quad} \end{bmatrix}$$

**Solution:** (Instructor solution preview: show the student solution after due date.)

**SOLUTION:**

A basis for the column space, determined from the pivot columns 1 and 2, is

$$\left\{ \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \right\}$$

Solve  $Ax = 0$ , to obtain  $x = s_1 \begin{bmatrix} +4 \\ -5 \\ 1 \\ 0 \end{bmatrix} + s_2 \begin{bmatrix} +3 \\ +1 \\ 0 \\ 1 \end{bmatrix}$ , and so

$$\text{the nullspace basis is } \left\{ \begin{bmatrix} +4 \\ -5 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} +3 \\ +1 \\ 0 \\ 1 \end{bmatrix} \right\}.$$

*Correct Answers:*

- $\begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}$
- $\begin{bmatrix} 2 \\ 3 \\ 6 \end{bmatrix}$
- $\begin{bmatrix} -3 \\ -3 \\ 1 \end{bmatrix}$

**14. (1 pt) Library/Rochester/setLinearAlgebra10Bases/ur\_la\_10.30.pg**

Find a basis of the column space of the matrix

$$A = \begin{bmatrix} -2 & -2 & 0 & -1 \\ 0 & 2 & -2 & -2 \\ -2 & -2 & 0 & -1 \end{bmatrix}.$$

$$\begin{bmatrix} \underline{\quad} \\ \underline{\quad} \\ \underline{\quad} \end{bmatrix}, \begin{bmatrix} \underline{\quad} \\ \underline{\quad} \\ \underline{\quad} \end{bmatrix}.$$

*Correct Answers:*

- $\begin{bmatrix} 0 \\ -2 \\ 0 \end{bmatrix}$
- $\begin{bmatrix} -2 \\ 0 \\ -2 \end{bmatrix}$
- $\begin{bmatrix} 0 \\ 0 \\ -2 \end{bmatrix}$

**15. (1 pt) Library/WHFreeman/Holt\_linear\_algebra/Chaps\_1-4-4.1.27.pg**

Find the null space for  $A = \begin{bmatrix} 1 & 1 \\ -7 & -6 \\ -6 & -2 \end{bmatrix}$ .

What is  $\text{null}(A)$ ?

- A.  $\text{span}\left\{\begin{bmatrix} -1 \\ 1 \end{bmatrix}\right\}$
- B.  $\mathbb{R}^3$
- C.  $\text{span}\left\{\begin{bmatrix} 1 \\ -7 \\ -6 \end{bmatrix}\right\}$
- D.  $\mathbb{R}^2$
- E.  $\text{span}\left\{\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}\right\}$
- F.  $\text{span}\left\{\begin{bmatrix} 1 \\ 1 \end{bmatrix}\right\}$
- G.  $\left\{\begin{bmatrix} 0 \\ 0 \end{bmatrix}\right\}$
- H. none of the above

**Solution:** (Instructor solution preview: show the student solution after due date.)

SOLUTION

$A$  is row reduced to  $\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$ . The basis of the null space has

one element for each column without a leading one in the row reduced matrix.

Thus  $\mathbf{Ax} = \mathbf{0}$  has a zero dimensional null space,

and  $\text{null}(A)$  is the zero vector  $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ .

Correct Answers:

- G

#### 16. (1 pt) local/Library/UI/4.1.23.pg

Find the null space for  $A = \begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & 8 \end{bmatrix}$ .

What is  $\text{null}(A)$ ?

- A.  $\text{span}\left\{\begin{bmatrix} -8 \\ +3 \end{bmatrix}\right\}$
- B.  $\text{span}\left\{\begin{bmatrix} -8 \\ +3 \\ 1 \end{bmatrix}\right\}$
- C.  $\mathbb{R}^2$
- D.  $\mathbb{R}^3$
- E.  $\text{span}\left\{\begin{bmatrix} 1 \\ 0 \\ +3 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -8 \end{bmatrix}\right\}$
- F.  $\text{span}\left\{\begin{bmatrix} +3 \\ -8 \\ 1 \end{bmatrix}\right\}$
- G.  $\text{span}\left\{\begin{bmatrix} +3 \\ -8 \end{bmatrix}\right\}$
- H. none of the above

**Solution:** (Instructor solution preview: show the student solution after due date.)

SOLUTION

$A$  is row reduced. The basis of the null space has one element for each column without a leading one in the row reduced matrix.

Thus  $\mathbf{Ax} = \mathbf{0}$  has a one dimensional null space,

and thus,  $\text{null}(A)$  is the subspace generated by  $\begin{bmatrix} 1 & -3 \\ 18 & 1 \end{bmatrix}$ .

Correct Answers:

- F

#### 17. (1 pt) Library/WHFreeman/Holt\_linear\_algebra/Chaps\_1-4-/4.3.47.pg

Indicate whether the following statement is true or false.

- [?] 1. If  $A$  and  $B$  are equivalent matrices, then  $\text{col}(A) = \text{col}(B)$ .

**Solution:** (Instructor solution preview: show the student solution after due date.)

SOLUTION:

FALSE. Consider  $A = \begin{bmatrix} 2 & 3 & 4 \\ 2 & 3 & 4 \end{bmatrix}$  and  $B = \begin{bmatrix} 2 & 3 & 4 \\ 0 & 0 & 0 \end{bmatrix}$

Correct Answers:

- F

#### 18. (1 pt) local/Library/UI/Fall14/HW7\_27.pg

Determine the rank and nullity of the matrix.

$$\begin{bmatrix} 2 & -1 & 0 & 1 \\ 5 & 2 & 1 & -4 \\ -1 & -4 & -1 & 6 \\ -8 & -5 & -2 & 9 \end{bmatrix}$$

The rank of the matrix is

- A. -4
- B. -3
- C. -2
- D. -1
- E. 0
- F. 1
- G. 2
- H. 3
- I. 4
- J. none of the above

The nullity of the matrix is

- A. -4
- B. -3
- C. -2
- D. -1
- E. 0

- F. 1
- G. 2
- H. 3
- I. 4
- J. none of the above

**Solution:** (*Instructor solution preview: show the student solution after due date.* )

**SOLUTION:**

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When reduced to row-echelon form, there are two non-zero rows, so the rank of the matrix is 2 and the nullity is 2.

$$\left[ \begin{array}{cccc} 2 & -1 & 0 & 1 \\ 5 & 2 & 1 & -4 \\ -1 & -4 & -1 & 6 \\ -8 & -5 & -2 & 9 \end{array} \right] \sim \left[ \begin{array}{cccc} 1 & 4 & 1 & -6 \\ 0 & 9 & 2 & -13 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

*Correct Answers:*

- G
- G