1. (1 pt) local/Library/UI/Fall14/HW7_4.pg

Determine if the subset of \mathbb{R}^2 consisting of vectors of the form $\begin{bmatrix} a \\ b \end{bmatrix}$, where *a* and *b* are integers, is a subspace.

Select true or false for each statement.

The set contains the zero vector

- A. True
- B. False

This set is closed under vector addition

- A. True
- B. False

This set is closed under scalar multiplications

- A. True
- B. False

This set is a subspace

- A. True
- B. False

Solution: (*Instructor solution preview: show the student solution after due date.*)

SOLUTION The vector $\begin{bmatrix} 1\\0 \end{bmatrix}$ is included in the set, but the vector $(1/2) * \begin{bmatrix} 1\\0 \end{bmatrix} = \begin{bmatrix} 1/2\\0 \end{bmatrix}$ is not included in the set. *Correct Answers:* • A • B • B

2. (1 pt) local/Library/UI/Fall14/HW7_5.pg

Determine if the subset of \mathbb{R}^3 consisting of vectors of the $\begin{bmatrix} a \end{bmatrix}$

form $\begin{vmatrix} b \\ c \end{vmatrix}$, where $a \ge 0, b \ge 0$, and $c \ge 0$ is a subspace.

Select true or false for each statement.

The set contains the zero vector

- A. True
- B. False

This set is closed under vector addition

- A. True
- B. False

This set is closed under scalar multiplications

• A. True

• B. False

This set is a subspace

- A. True
- B. False

Solution: (*Instructor solution preview: show the student solution after due date.*)

SOLUTION The vector $\begin{bmatrix} 1\\0\\0 \end{bmatrix}$ is included in the set, but the vector $(-1)*\begin{bmatrix} 1\\0\\0 \end{bmatrix} = \begin{bmatrix} -1\\0\\0 \end{bmatrix}$ is not included in the set. *Correct Answers:* • A • B • B • B

3. (1 pt) local/Library/UI/Fall14/HW7_6.pg

If *A* is an $n \times n$ matrix and $\mathbf{b} \neq 0$ in \mathbb{R}^n , then consider the set of solutions to $A\mathbf{x} = \mathbf{b}$.

Select true or false for each statement.

The set contains the zero vector

- A. True
- B. False

This set is closed under vector addition

- A. True
- B. False

This set is closed under scalar multiplications

- A. True
- B. False

This set is a subspace

- A. True
- B. False

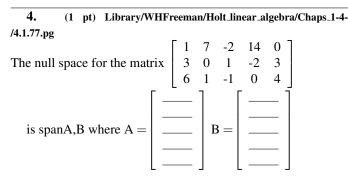
Solution: (*Instructor solution preview: show the student solution after due date.*)

SOLUTION

 $A\mathbf{0} = \mathbf{0} \neq \mathbf{b}$, so the zero vector is not in the set and it is not a subspace.

Correct Answers:

- B
- B
- B
- B



Solution: (*Instructor solution preview: show the student solution after due date.*)

SOLUTION:

We can use a CAS to get

. -

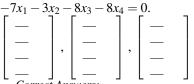
Correct Answers:

- 0.428571428571429
- -1.85714285714286
- 0.714285714285714
- 1
- 0

• -0.767857142857143

- 0.0892857142857143
- -0.696428571428571
- 0
- 1

5. (1 pt) Library/Rochester/setLinearAlgebra10Bases/ur_la_10_26.pg Find a basis of the subspace of \mathbb{R}^4 defined by the equation



Correct Answers:

• \(\displaystyle\left.\begin{array}{c}
 \mbox{-3} \cr
 \mbox{7} \cr
 \mbox{0} \cr
 \mbox{0} \cr
 \end{array}\right.\),\(\displaystyle\]
 \mbox{-8} \cr
 \mbox{0} \cr
 \mbox{7} \cr

\end{array}\right.\)

6. (1 pt) local/Library/UI/6a.pg				
	2	0	5	
	-1	6	2	
6. (1 pt) local/Library/UI/6a.pg The null space for the matrix	4	4	-1	
	5	1	0	
	4	1	1	
	-			



Solution: (*Instructor solution preview: show the student solution after due date.*)

SOLUTION:

$$\begin{bmatrix} -0.767857142857143\\ 0.6892857142857143\\ -0.69642857142857143\\ -0.696428571428571\\ 4 & 4 & 0^{-1}\\ 5 & 1 & 10\\ 4 & 1 & 1 \end{bmatrix} = \left\{ \begin{bmatrix} 0\\ 0\\ 0\\ 0 \end{bmatrix} \right\}$$

Correct Answers:
• 0
• 0
• 0
• 0
• 0
• 7. (1 pt) Library/WHFreeman/Holt_linear_algebra/Chaps_1-4-
/4.2.32a.pg
Find a basis for the null space of matrix A.

Solution: (*Instructor solution preview: show the student solution after due date.*)

SOLUTION:

Row-reduce the matrix which has the given vectors as columns.

 $\frac{|ar_{A}a_{A}|^{2}}{ar_{A}a_{A}} = 0 \ \text{has solutions of the } \frac{|ar_{A}a_{A}|^{2}}{ar_{A}a_{A}} = 0 \ \text{has solutions of the } \frac{|ar_{A}a_{A}|^{2}}{ar_{A}a_{A}} = 0 \ \text{has solutions of the } \frac{|ar_{A}a_{A}|^{2}}{ar_{A}a_{A}} = 0 \ \text{has solutions of the } \frac{|ar_{A}a_{A}|^{2}}{ar_{A}a_{A}} = 0 \ \text{has solutions of the } \frac{|ar_{A}a_{A}|^{2}}{ar_{A}a_{A}} = 0 \ \text{has solutions of the } \frac{|ar_{A}a_{A}|^{2}}{ar_{A}a_{A}} = 0 \ \text{has solutions of the } \frac{|ar_{A}a_{A}|^{2}}{ar_{A}a_{A}} = 0 \ \text{has solutions of the } \frac{|ar_{A}a_{A}|^{2}}{ar_{A}a_{A}} = 0 \ \text{has solutions of the } \frac{|ar_{A}a_{A}|^{2}}{ar_{A}a_{A}} = 0 \ \text{has solutions of the } \frac{|ar_{A}a_{A}|^{2}}{ar_{A}a_{A}} = 0 \ \text{has solutions of } \frac{|ar_{A}a_{A}|^{2}}{ar_{A}a_{A}} = 0 \ \text{has solutions } \frac{|ar_{A}a_{A}|^{2}}{ar_{A}a_{A}} = 0 \ \text{has s$

(IIIDOX { 0 } / CL					
\mbox{7} \cr		4		14]	
<pre>\mbox{0} \cr</pre>		0		14 3	
<pre>\end{array}\right.\) ,\(\displaystyle\left.</pre>	array}{c}	1			
\mbox{-8} \cr	$\mathbf{x} = s_1$	1	$+s_{2}$	0 -3	
<pre>\mbox{0} \cr</pre>		0		-3	
\mbox{0} \cr		0		1	
\mbox{7} \cr	so that a basis for the subspace is				

$\begin{cases} \left[\begin{array}{c} 4\\0\\1\\0\end{array}\right], \left[\begin{array}{c} 14\\3\\0\\-3\\1\end{array}\right] \right\}$ Correct Answers: $ \cdot \left\{ \left(\text{displaystyle} \text{left.} \text{begin} \{ \text{array} \} \{ c \} \right. \\ \left. \text{whox}\{4 \setminus \text{cr} \\ \text{whox}\{4 \setminus \text{cr} \\ \text{whox}\{1 \in \text{cr} \\ \text{whox}\{1\} \in \text{cr} \\ \text{whox}\{0\} \in \text{cr} \\ \text{whox}\{0\} \in \text{cr} \\ \text{whox}\{3\} = \text{cr} \\ \text$	• J. none of the above Solution: (Instructor solution preview: show the student solution after due date.) SOLUTION: Row reduce A to get: $\begin{bmatrix} 2 & 1 & 0 & 7 \\ -2 & 2 & x & -7 \\ 3 & 7 & 4 & 28 \end{bmatrix} \sim \begin{bmatrix} 2 & 1 & 0 & 7 \\ 0 & 4 & x & 7 \\ 0 & 12 & 12 & 21 \end{bmatrix}$ Since two pivots are needed, $x = 4$ Correct Answers: • I array/fel 10. (1 p) local/Library/Ul/Fal11/HW7.12.pg Suppose that A is a 8 × 6 matrix which has a null space of dimension 6. The rank of A= • A4 • B3 • C2 • D1 • E. 0 • F. 1 • G. 2 • H. 3 • I. 4 • J. none of the above Solution: (Instructor solution preview: show the student solution after due date.) SOLUTION Using the Rank-Nullity theorem, if the dimensions of A is n x m, rank(A) = m - nullity(A) = 6 - 6 = 0 Correct Answers: • E Suppose A is a 5 × 4 matrix. If rank of $A = 1$, then nullity of $A =$ • A4 B3 • C2 • D1 E. 0 • F. 1 • E. 0 • F. 1 • C. 2 • D1 • E. 0 • F. 1 • G. 2
• A4	 E. 0 F. 1 G. 2 H. 3 I. 4 J. none of the above <i>Correct Answers:</i> H

The vector \vec{b} is NOT in *ColA* if and only if $A\vec{v} = \vec{b}$ does NOT have a solution

- A. True
- B. False

Correct Answers:

• A

The vector \vec{b} is in *ColA* if and only if $A\vec{v} = \vec{b}$ has a solution

- A. True
- B. False

Correct Answers:

• A

The vector \vec{v} is in *NulA* if and only if $A\vec{v} = \vec{0}$

- A. True
- B. False

Correct Answers:

• A

If the equation $A\vec{x} = \vec{b_1}$ has at least one solution and if the equation $A\vec{x} = \vec{b_2}$ has at least one solution, then the equation $A\vec{x} = -1\vec{b_1} - 3\vec{b_2}$ also has at least one solution.

- A. True
- B. False

Correct Answers:

• A

If $\vec{x_1}$ and $\vec{x_2}$ are solutions to $A\vec{x} = \vec{0}$, then $8\vec{x_1} - 1\vec{x_2}$ is also a solution to $A\vec{x} = \vec{0}$.

• A. True

• B. False

Correct Answers:

• A

If $\vec{x_1}$ and $\vec{x_2}$ are solutions to $A\vec{x} = \vec{b}$, then $2\vec{x_1} - 9\vec{x_2}$ is also a solution to $A\vec{x} = \vec{b}$.

• A. True

• B. False

Correct Answers:

• B

Suppose *A* is a 4×2 matrix. Then *nul A* is a subspace of R^k where k =

• A. -4

- B. -3
- C. -2
- D. -1 • E. 0
- E. 0
- G. 2
- G. 2 • H. 3
- I. 4
- J. none of the above

Correct Answers:

• G

Suppose *A* is a 2 × 6 matrix. Then *col A* is a subspace of R^k where k =

- A. -4
- B. -3
- C. -2
- D. -1 • E. 0
- E. 0 • F. 1
- G. 2
- H. 3
- I. 4
- J. none of the above

```
Correct Answers:
```

• G

20. (1 pt) Library/TCNJ/TCNJ_BasesLinearlyIndependentSet-/problem5.pg

	[1]		1		1	
Let W_1 be the set:	0	,	1	,	1	.
Let W_1 be the set:	0		0		1	

Determine if W_1 is a basis for \mathbb{R}^3 and check the correct answer(s) below.

- A. W_1 is a basis.
- B. W_1 is not a basis because it does not span \mathbb{R}^3 .
- C. W_1 is not a basis because it is linearly dependent.

Let W_2 be the set: $\begin{bmatrix} 1\\0\\1 \end{bmatrix}$, $\begin{bmatrix} 0\\0\\0 \end{bmatrix}$, $\begin{bmatrix} 0\\1\\0 \end{bmatrix}$.

Determine if W_2 is a basis for \mathbb{R}^3 and check the correct answer(s) below.

- A. W_2 is not a basis because it does not span \mathbb{R}^3 .
- B. W_2 is a basis.

• C. W_2 is not a basis because it is linearly dependent.

Correct Answers:

AAC

21. (1 pt) local/Library/UI/Fall14/HW7_25.pg Indicate whether the following statement is true or false? If $S = \text{span}u_1, u_2, u_3$, then dim(S) = 3.

- A. True
- B. False

Solution: (*Instructor solution preview: show the student solution after due date.*)

SOLUTION: FALSE. For example, suppose $S = \text{span}\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\},$ then dim(S) < 3 *Correct Answers:* • B

22. (1 pt) local/Library/TCNJ/TCNJ_BasesLinearlyIndependentSet-/3.pg

Check the true statements below:

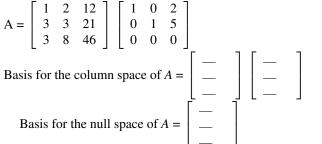
- A. The columns of an invertible *n* × *n* matrix form a basis for ℝⁿ.
- B. If $H = Span\{b_1, ..., b_p\}$, then $\{b_1, ..., b_p\}$ is a basis for H.
- C. If *B* is an echelon form of a matrix *A*, then the pivot columns of *B* form a basis for *ColA*.
- D. The column space of a matrix *A* is the set of solutions of *Ax* = *b*.

• E. A basis is a spanning set that is as large as possible. *Correct Answers:*

• A

23. (1 pt) local/Library/UI/4.3.1a.pg

Find bases for the column space and the null space of matrix A. You should verify that the Rank-Nullity Theorem holds. An equivalent echelon form of matrix A is given to make your work easier.



Solution: (*Instructor solution preview: show the student solution after due date.*)

SOLUTION: A basis for the column space, determined from the pivot columns 1 and 2, is 3 3 3 Solve $A\mathbf{x} = \mathbf{0}$, to obtain $\mathbf{x} = s \begin{bmatrix} 2 \\ 5 \\ 1 \end{bmatrix}$, and so the nullspace 5 basis is Correct Answers: • \(\displaystyle\left.\begin{array}{c} \mbox{1} \cr \mbox{3} \cr \mbox{3} \cr \end{array}\right.\) , \ (\displaystyle\left.\begin{array}{c \mbox{2} \cr \mbox{3} \cr \mbox{8} \cr \end{array}\right.\) \(\displaystyle\left.\begin{array}{c} \mbox{2} \cr \mbox{5} \cr $mbox{1} \cr$ \end{array}\right.\)

24. (1 pt) Library/WHFreeman/Holt_linear_algebra/Chaps_1-4-/4.1.22.pg

Find the null space for $A = \begin{bmatrix} 3 & 2 \\ 1 & -9 \end{bmatrix}$. What is null(A)?

• A. span
$$\left\{ \begin{bmatrix} 1 \\ 3 \end{bmatrix} \right\}$$

• B. span $\left\{ \begin{bmatrix} 3 \\ 1 \end{bmatrix} \right\}$
• C. \mathbb{R}^2
• D. span $\left\{ \begin{bmatrix} -2 \\ 3 \end{bmatrix} \right\}$
• E. $\left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\}$
• F. span $\left\{ \begin{bmatrix} -1 \\ 3 \end{bmatrix} \right\}$
• G. span $\left\{ \begin{bmatrix} 3 \\ 2 \end{bmatrix} \right\}$
• H. none of the above

Solution: (*Instructor solution preview: show the student solution after due date.*)

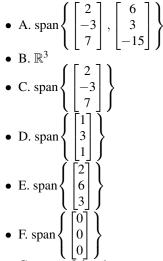
SOLUTION

A row reduces to the identity matrix.

Thus $A\mathbf{x} = \mathbf{0}$ has only the trivial solution $\begin{bmatrix} 0\\0 \end{bmatrix}$, and thus, null(A) = $\begin{bmatrix} 0\\0 \end{bmatrix}$. *Correct Answers:* \bullet E

25. (1 pt) Library/WHFreeman/Holt_linear_algebra/Chaps_1-4-/4.1.30.pg

Find the null space for $A = \begin{bmatrix} 2 & -3 & 7 \\ 6 & 3 & -15 \\ 3 & 5 & -18 \end{bmatrix}$. What is null(A)?



• G. none of the above

Solution: (*Instructor solution preview: show the student solution after due date.*)

SOLUTION A is row reduces to $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix}$. The basis of the null space

has one element for each column without a leading one in the row reduced matrix.

Thus $A\mathbf{x} = \mathbf{0}$ has a one dimentional null space, and null(A) is the subspace generated by $\begin{bmatrix} 1\\3\\1 \end{bmatrix}$.

Correct Answers:

• D

26. (1 pt) Library/WHFreeman/Holt_linear_algebra/Chaps_1-4-/4.1.28.pg

Find the null space for $A = \begin{bmatrix} 2 & 6 \\ 7 & 21 \\ 4 & 12 \end{bmatrix}$.

What is null(A)?

• A.
$$\mathbb{R}^{3}$$

• B. span $\left\{ \begin{bmatrix} 0\\0\\0 \end{bmatrix} \right\}$
• C. \mathbb{R}^{2}
• D. span $\left\{ \begin{bmatrix} 12\\4 \end{bmatrix} \right\}$
• E. span $\left\{ \begin{bmatrix} -3\\1 \end{bmatrix} \right\}$
• F. span $\left\{ \begin{bmatrix} 2\\7\\4 \end{bmatrix} \right\}$
• G. span $\left\{ \begin{bmatrix} 0\\0 \end{bmatrix} \right\}$
• H. none of the above

Solution: (*Instructor solution preview: show the student solution after due date.*)

SOLUTION
A is row reduces to
$$\begin{bmatrix} 2 & 6 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$
. The basis of the null space has

one element for each column without a leading one in the row reduced matrix.

Thus $A\mathbf{x} = \mathbf{0}$ has a one dimentional null space,

and null(A) is the subspace generated by $\begin{bmatrix} -6\\ 2 \end{bmatrix}$. *Correct Answers:*

27. (1 pt) local/Library/UI/4.3.3.pg

Find bases for the column space and the null space of matrix A. You should verify that the Rank-Nullity Theorem holds. An equivalent echelon form of matrix A is given to make your work easier.

Solution: (*Instructor solution preview: show the student solution after due date.*)

SOLUTION:

6

A basis for the column space, determined from the pivot columns 1 and 2, is

$$\left\{ \begin{bmatrix} 1\\ -2\\ 0 \end{bmatrix}, \begin{bmatrix} 0\\ 1\\ 1 \end{bmatrix} \right\}$$

Solve $A\mathbf{x} = \mathbf{0}$, to obtain $\mathbf{x} = s_1 \begin{bmatrix} +5\\ -5\\ 1\\ 0 \end{bmatrix} + s_2 \begin{bmatrix} +4\\ +1\\ 0\\ 1 \end{bmatrix}$, and so
the nullspace basis is $\left\{ \begin{bmatrix} +5\\ -5\\ 1\\ 0 \end{bmatrix}, \begin{bmatrix} +4\\ +1\\ 0\\ 1 \end{bmatrix} \right\}$.
Correct Answers:
• \(\displaystyle\left.\begin{array}{c}

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 $mbox{1} \cr$

```
mbox{-2} \ cr
 \mbox{0} \cr
  \end{array}\right.\) ,\(\displaystyle\left.\begin{array}{c
  \mbox{0} \cr
 mbox{1} \cr
 mbox{1} \cr
 \end{array}\right.\)
• \(\displaystyle\left.\begin{array}{c}
 mbox{5} \cr
 mbox{-5} \cr
 mbox{1} \cr
 \mbox{0} \cr
 \end{array}\right.\) ,\(\displaystyle\left.\begin{array}{c
 \mbox{4} \cr
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  \end{array}\right.\)
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