
1. (1 pt) Library/WHFreeman/Holt_linear_algebra/Chaps.1-4/2.3.40.pg

Let S be a set of m vectors in \mathbb{R}^n with $m > n$.
Select the best statement.

- A. The set S is linearly independent, as long as it does not include the zero vector.
- B. The set S is linearly dependent.
- C. The set S is linearly independent, as long as no vector in S is a scalar multiple of another vector in the set.
- D. The set S is linearly independent.
- E. The set S could be either linearly dependent or linearly independent, depending on the case.
- F. none of the above

Solution: (Instructor solution preview: show the student solution after due date.)

SOLUTION

By theorem 2.13, a linearly independent set in \mathbb{R}^n can contain no more than n vectors.

Correct Answers:

- B

2. (1 pt) Library/WHFreeman/Holt_linear_algebra/Chaps.1-4/2.3.41.pg

Let A be a matrix with more rows than columns.
Select the best statement.

- A. The columns of A are linearly independent, as long as no column is a scalar multiple of another column in A
- B. The columns of A could be either linearly dependent or linearly independent depending on the case.
- C. The columns of A are linearly independent, as long as they does not include the zero vector.
- D. The columns of A must be linearly dependent.
- E. The columns of A must be linearly independent.
- F. none of the above

Solution: (Instructor solution preview: show the student solution after due date.)

SOLUTION

The zero matrix is an example where the columns are linearly dependent. The matrix where the top square portion is the identity matrix and the portion below that is all zeros is an example where the columns are linearly independent.

The columns of A could be either linearly dependent or linearly independent depending on the case.

Correct Answers:

- B

3. (1 pt) Library/WHFreeman/Holt_linear_algebra/Chaps.1-4/2.3.42.pg

Let A be a matrix with more columns than rows.
Select the best statement.

- A. The columns of A are linearly independent, as long as they does not include the zero vector.
- B. The columns of A could be either linearly dependent or linearly independent depending on the case.
- C. The columns of A must be linearly dependent.
- D. The columns of A are linearly independent, as long as no column is a scalar multiple of another column in A
- E. none of the above

Solution: (Instructor solution preview: show the student solution after due date.)

SOLUTION

Since there are more columns than rows, when we row reduce the matrix not all columns can have a leading 1.

The columns of A must be linearly dependent.

Correct Answers:

- C

4. (1 pt) local/Library/UI/MatrixAlgebra/Euclidean/2.3.43.pg

Let A be a matrix with linearly independent columns.
Select the best statement.

- A. The equation $A\mathbf{x} = \mathbf{0}$ always has nontrivial solutions.
- B. There is insufficient information to determine if such an equation has nontrivial solutions.
- C. The equation $A\mathbf{x} = \mathbf{0}$ has nontrivial solutions precisely when it has more rows than columns.
- D. The equation $A\mathbf{x} = \mathbf{0}$ never has nontrivial solutions.
- E. The equation $A\mathbf{x} = \mathbf{0}$ has nontrivial solutions precisely when it has more columns than rows.
- F. The equation $A\mathbf{x} = \mathbf{0}$ has nontrivial solutions precisely when it is a square matrix.
- G. none of the above

Solution: (Instructor solution preview: show the student solution after due date.)

SOLUTION

The linear independence of the columns does not change with row reduction. Since the columns are linearly independent, after row reduction, each column contains a leading 1. We get nontrivial solutions when we have columns without a leading 1 in the row reduced matrix.

The equation $A\mathbf{x} = \mathbf{0}$ never has nontrivial solutions.

Correct Answers:

- D

5. (1 pt) local/Library/UI/MatrixAlgebra/Euclidean/2.3.44.pg

Let A be a matrix with linearly independent columns. Select the best statement.

- A. The equation $A\mathbf{x} = \mathbf{b}$ has a solution for all \mathbf{b} precisely when it has more columns than rows.
- B. The equation $A\mathbf{x} = \mathbf{b}$ always has a solution for all \mathbf{b} .
- C. The equation $A\mathbf{x} = \mathbf{b}$ has a solution for all \mathbf{b} precisely when it is a square matrix.
- D. The equation $A\mathbf{x} = \mathbf{b}$ never has a solution for all \mathbf{b} .
- E. There is insufficient information to determine if $A\mathbf{x} = \mathbf{b}$ has a solution for all \mathbf{b} .
- F. The equation $A\mathbf{x} = \mathbf{b}$ has a solution for all \mathbf{b} precisely when it has more rows than columns.
- G. none of the above

Solution: (*Instructor solution preview: show the student solution after due date.*)

SOLUTION

A linear equation has a solution when the row reduced form of the augmented solution does not have a leading 1 in the extra column that corresponds to constants. Since the columns of the matrix are linearly independent the number of columns is no more than the number of rows. If there are fewer columns than rows we can produce a \mathbf{b} for which there is no solution.

The equation $A\mathbf{x} = \mathbf{b}$ has a solution for all \mathbf{b} precisely when it is a square matrix.

Correct Answers:

- C

6. (1 pt) Library/WHFreeman/Holt_linear_algebra/Chaps.1-4/2.3.46.pg

Let $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ be a linearly dependent set of vectors. Select the best statement.

- A. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ is a linearly independent set of vectors unless $\mathbf{u}_4 = \mathbf{0}$.
- B. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ could be a linearly independent or linearly dependent set of vectors depending on the vectors chosen.
- C. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ is always a linearly independent set of vectors.

- D. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ is a linearly independent set of vectors unless \mathbf{u}_4 is a linear combination of other vectors in the set.
- E. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ is always a linearly dependent set of vectors.
- F. none of the above

Solution: (*Instructor solution preview: show the student solution after due date.*)

SOLUTION

If the zero vector is a nontrivial linear combination of a vectors in a smaller set, then it is also a nontrivial combination of vectors in a bigger set containing those vectors.

$\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ is always a linearly dependent set of vectors.

Correct Answers:

- E

7. (1 pt) Library/WHFreeman/Holt_linear_algebra/Chaps.1-4/2.3.47.pg

Let $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ be a linearly independent set of vectors. Select the best statement.

- A. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ is never a linearly independent set of vectors.
- B. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ could be a linearly independent or linearly dependent set of vectors depending on the vectors chosen.
- C. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ is always a linearly independent set of vectors.
- D. none of the above

Solution: (*Instructor solution preview: show the student solution after due date.*)

SOLUTION

If the zero vector cannot be written as a nontrivial linear combination of a vectors in a smaller set, then it is also not a nontrivial combination of vectors in a proper subset of those vectors.

$\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ is always a linearly independent set of vectors.

Correct Answers:

- C

8. (1 pt) Library/WHFreeman/Holt_linear_algebra/Chaps.1-4/2.3.49.pg

Let \mathbf{u}_4 be a linear combination of $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$. Select the best statement.

- A. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ could be a linearly dependent or linearly independent set of vectors depending on the vectors chosen.
- B. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ is a linearly dependent set of vectors unless one of $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ is the zero vector.
- C. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ is always a linearly independent set of vectors.

- D. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ is never a linearly independent set of vectors.
- E. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ is never a linearly dependent set of vectors.
- F. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ could be a linearly dependent or linearly independent set of vectors depending on the vector space chosen.
- G. none of the above

Solution: (Instructor solution preview: show the student solution after due date.)

SOLUTION

If $\mathbf{u}_4 = a_1\mathbf{u}_1 + a_2\mathbf{u}_2 + a_3\mathbf{u}_3$, then

$$0 = a_1\mathbf{u}_1 + a_2\mathbf{u}_2 + a_3\mathbf{u}_3 - \mathbf{u}_4$$

" $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ is never a linearly independent set of vectors."

Correct Answers:

- D

9. (1 pt) Library/Rochester/setLinearAlgebra9Dependence/ur.la.9.7.pg

The vectors

$$v = \begin{bmatrix} -1 \\ 0 \\ 3 \end{bmatrix}, u = \begin{bmatrix} -4 \\ -12 \\ 31+k \end{bmatrix}, \text{ and } w = \begin{bmatrix} -3 \\ -7 \\ 16 \end{bmatrix}.$$

are linearly independent if and only if $k \neq$ ____.

Correct Answers:

- -7

10. (1 pt) Library/TCNJ/TCNJ.LinearIndependence/problem3.pg

If k is a real number, then the vectors $(1, k), (k, 3k + 40)$ are linearly independent precisely when

$$k \neq a, b,$$

where $a =$ ____, $b =$ ____, and $a < b$.

Correct Answers:

- -5
- 8

11. (1 pt) Library/Rochester/setLinearAlgebra4InverseMatrix/ur.la.4.2.pg

The matrix $\begin{bmatrix} 4 & -5 \\ -6 & k \end{bmatrix}$ is invertible if and only if $k \neq$ ____.

Correct Answers:

- 7.5

12. (1 pt) Library/Rochester/setAlgebra34Matrices/scalarmult3.pg

If $A = \begin{bmatrix} 1 & -2 & 1 \\ 0 & -4 & -1 \\ 3 & -1 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & -3 & 3 \\ 1 & -2 & 2 \\ 1 & -1 & 2 \end{bmatrix}$, then

$$2A - 3B = \begin{bmatrix} _ & _ & _ \\ _ & _ & _ \\ _ & _ & _ \end{bmatrix} \text{ and}$$

$$A^T = \begin{bmatrix} _ & _ & _ \\ _ & _ & _ \\ _ & _ & _ \end{bmatrix}.$$

Correct Answers:

- $\begin{bmatrix} -10 & 5 & -7 \\ -3 & -2 & -8 \\ 3 & 1 & -10 \end{bmatrix}$
- $\begin{bmatrix} 1 & 0 & 3 \\ -2 & -4 & -1 \\ 1 & -1 & -2 \end{bmatrix}$

13. (1 pt) Library/Rochester/setAlgebra34Matrices/cubing_2x2.pg

Given the matrix $A = \begin{bmatrix} 4 & -3 \\ 0 & 3 \end{bmatrix}$, find A^3 .

$$A^3 = \begin{bmatrix} _ & _ \\ _ & _ \end{bmatrix}.$$

Correct Answers:

- 64
- -111
- 0
- 27

14. (1 pt) Library/maCalcDB/setLinearAlgebra3Matrices/ur.la.3.6.pg

If A and B are 6×2 matrices, and C is a 4×6 matrix, which of the following are defined?

- A. $C + B$
- B. CA
- C. $B + A$
- D. B^T
- E. $B^T C^T$
- F. AB

Correct Answers:

- BCDE

15. (1 pt) Library/NAU/setLinearAlgebra/m1.pg

Find the inverse of AB if

$$A^{-1} = \begin{bmatrix} 3 & -2 \\ 3 & 5 \end{bmatrix} \text{ and } B^{-1} = \begin{bmatrix} 2 & -3 \\ 2 & -1 \end{bmatrix}.$$

$$(AB)^{-1} = \begin{bmatrix} _ & _ \\ _ & _ \end{bmatrix}$$

Correct Answers:

- -3
- -19
- 3
- -9

16. (1 pt) Library/Rochester/setLinearAlgebra4InverseMatrix/ur.Ch2.1.4.pg

Are the following matrices invertible? Enter "Y" or "N".
You must get all of the answers correct to receive credit.

- 1. $\begin{bmatrix} -5 & 0 \\ 0 & 5 \end{bmatrix}$
 —2. $\begin{bmatrix} 25 & -8 \\ 0 & 0 \end{bmatrix}$
 —3. $\begin{bmatrix} -7 & 6 \\ -1 & -2 \end{bmatrix}$
 —4. $\begin{bmatrix} -5 & -8 \\ 25 & 40 \end{bmatrix}$

Correct Answers:

- Y
- N
- Y
- N

17. (1 pt) Library/ASU-topics/set119MatrixAlgebra/p13.pg

Consider the following two systems.

(a)

$$\begin{cases} x - 3y = 2 \\ -3x + 3y = -3 \end{cases}$$

(b)

$$\begin{cases} x - 3y = 3 \\ -3x + 3y = -3 \end{cases}$$

(i) Find the inverse of the (common) coefficient matrix of the two systems.

$$A^{-1} = \begin{bmatrix} \underline{\hspace{1cm}} & \underline{\hspace{1cm}} \\ \underline{\hspace{1cm}} & \underline{\hspace{1cm}} \end{bmatrix}$$

(ii) Find the solutions to the two systems by using the inverse, i.e. by evaluating $A^{-1}B$ where B represents the right hand side

(i.e. $B = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$ for system (a) and $B = \begin{bmatrix} 3 \\ -3 \end{bmatrix}$ for system

(b)).

Solution to system (a): $x = \underline{\hspace{1cm}}$, $y = \underline{\hspace{1cm}}$

Solution to system (b): $x = \underline{\hspace{1cm}}$, $y = \underline{\hspace{1cm}}$

Correct Answers:

- -0.5
- -0.5
- -0.5
- -0.1666666666666667
- 0.5
- -0.5
- 0
- -1

18. (1 pt) Library/ASU-topics/set119MatrixAlgebra/p13.pg

Consider the following two systems.

(a)

$$\begin{cases} -2x + y = -3 \\ 3x - y = -3 \end{cases}$$

(b)

$$\begin{cases} -2x + y = -2 \\ 3x - y = -2 \end{cases}$$

(i) Find the inverse of the (common) coefficient matrix of the two systems.

$$A^{-1} = \begin{bmatrix} \underline{\hspace{1cm}} & \underline{\hspace{1cm}} \\ \underline{\hspace{1cm}} & \underline{\hspace{1cm}} \end{bmatrix}$$

(ii) Find the solutions to the two systems by using the inverse, i.e. by evaluating $A^{-1}B$ where B represents the right hand side

(i.e. $B = \begin{bmatrix} -3 \\ -3 \end{bmatrix}$ for system (a) and $B = \begin{bmatrix} -2 \\ -2 \end{bmatrix}$ for system

(b)).

Solution to system (a): $x = \underline{\hspace{1cm}}$, $y = \underline{\hspace{1cm}}$

Solution to system (b): $x = \underline{\hspace{1cm}}$, $y = \underline{\hspace{1cm}}$

Correct Answers:

- 1
- 1
- 3
- 2
- -6
- -15
- -4
- -10

19. (1 pt) Library/TCNJ/TCNJ_MatrixInverse/problem1.pg

If

$$A = \begin{bmatrix} -3 & 4 \\ 1 & 7 \end{bmatrix},$$

then

$$A^{-1} = \begin{bmatrix} \underline{\hspace{1cm}} & \underline{\hspace{1cm}} \\ \underline{\hspace{1cm}} & \underline{\hspace{1cm}} \end{bmatrix}.$$

Given $\vec{b} = \begin{bmatrix} 1 \\ -4 \end{bmatrix}$, solve $A\vec{x} = \vec{b}$.

$$\vec{x} = \begin{bmatrix} \underline{\hspace{1cm}} \\ \underline{\hspace{1cm}} \end{bmatrix}.$$

Correct Answers:

- $\begin{bmatrix} -0.28 & 0.16 \\ 0.04 & 0.12 \end{bmatrix}$
- $\begin{bmatrix} -0.92 \\ -0.44 \end{bmatrix}$

20. (1 pt) Library/WHFreeman/Holt_linear_algebra/Chaps.1-4/3.3.42.pg

A must be a square matrix to be invertible.

Solution: (Instructor solution preview: show the student solution after due date.)

SOLUTION: True, since

$$(B^{-1}A^{-1})(AB) = B^{-1}(A^{-1}A)B = B^{-1}I_nB = B^{-1}B = I_n.$$

Correct Answers:

- True

21. (1 pt) Library/Rochester/setLinearAlgebra4InverseMatrix/ur.la.4.11.pg

$$\text{If } A = \begin{bmatrix} -2e^{2t} \sin(4t) & -2e^{5t} \cos(4t) \\ -6e^{2t} \cos(4t) & 6e^{5t} \sin(4t) \end{bmatrix}$$

$$\text{then } A^{-1} = \begin{bmatrix} \underline{\hspace{2cm}} & \underline{\hspace{2cm}} \\ \underline{\hspace{2cm}} & \underline{\hspace{2cm}} \end{bmatrix}.$$

Correct Answers:

- $\sin(4t)/-2/2.71828182845905^{2t}$
- $-\cos(4t)/6/2.71828182845905^{2t}$
- $-\cos(4t)/2/2.71828182845905^{5t}$
- $\sin(4t)/6/2.71828182845905^{5t}$

22. (1 pt) Library/maCalcDB/setLinearAlgebra4InverseMatrix/ur.la.4.8.pg

Determine which of the formulas hold for all invertible $n \times n$ matrices A and B

- A. $ABA^{-1} = B$
- B. $(A+B)(A-B) = A^2 - B^2$
- C. A^7B^6 is invertible
- D. $A+B$ is invertible
- E. $(I_n - A)(I_n + A) = I_n - A^2$
- F. $(A + A^{-1})^9 = A^9 + A^{-9}$

Correct Answers:

- CE