

[12] 1.) The following matrices are all row equivalent. Use that information to fill in the 9 blanks below:

$$\begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} -8 & -6 & -4 & -38 \\ 12 & 5 & 11 & 43 \\ 8 & 10 & 0 & 50 \\ -12 & -1 & -17 & -27 \end{bmatrix} \sim \begin{bmatrix} 4 & 3 & 2 & 19 \\ 0 & -4 & 5 & -14 \\ 0 & 0 & -1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \sim$$

Determine the following linear combination relationships below (i.e., fill in the 9 blanks below):

$$\underline{5} \begin{bmatrix} -8 \\ 12 \\ 8 \\ -12 \end{bmatrix} + \underline{1} \begin{bmatrix} -6 \\ 5 \\ 10 \\ -1 \end{bmatrix} + \underline{-2} \begin{bmatrix} -4 \\ 11 \\ 0 \\ -17 \end{bmatrix} = \begin{bmatrix} -38 \\ 43 \\ 50 \\ -27 \end{bmatrix}$$

$$\underline{5} \begin{bmatrix} 4 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \underline{1} \begin{bmatrix} 3 \\ -4 \\ 0 \\ 0 \end{bmatrix} + \underline{-2} \begin{bmatrix} 2 \\ 5 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 19 \\ -14 \\ 2 \\ 0 \end{bmatrix}$$

$$\underline{5} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \underline{1} \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} + \underline{-2} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 5 \\ 1 \\ -2 \\ 0 \end{bmatrix}$$

[30] 2.) Solve the following systems of equations. Write your answer in parametric vector format.
SHOW YOUR WORK on NEXT PAGE.

$$2a.) \begin{bmatrix} 6 & 24 & -2 & -4 & -6 \\ 3 & 12 & 0 & 6 & -9 \\ 0 & 0 & 4 & 32 & -24 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -4 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} x_2 + \begin{bmatrix} -2 \\ 0 \\ -8 \\ 1 \\ 0 \end{bmatrix} x_4 + \begin{bmatrix} 3 \\ 0 \\ 6 \\ 0 \\ 1 \end{bmatrix} x_5$$

Answer: _____

$$2b.) \begin{bmatrix} 6 & 24 & -2 & -4 & -6 \\ 3 & 12 & 0 & 6 & -9 \\ 0 & 0 & 4 & 32 & -24 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 4 \\ 3 \\ 4 \end{bmatrix}$$

$$\begin{bmatrix} -4 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} x_2 + \begin{bmatrix} -2 \\ 0 \\ -8 \\ 1 \\ 0 \end{bmatrix} x_4 + \begin{bmatrix} 3 \\ 0 \\ 6 \\ 0 \\ 1 \end{bmatrix} x_5 + \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Answer: _____

$$2c.) \begin{bmatrix} 6 & 24 & -2 & -4 & -6 \\ 3 & 12 & 0 & 6 & -9 \\ 0 & 0 & 4 & 32 & -24 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

Answer: _____

no solution

Scratch work for 2a.) $\begin{bmatrix} 6 & 24 & -2 & -4 & -6 \\ 3 & 12 & 0 & 6 & -9 \\ 0 & 0 & 4 & 32 & -24 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

2b.) $\begin{bmatrix} 6 & 24 & -2 & -4 & -6 \\ 3 & 12 & 0 & 6 & -9 \\ 0 & 0 & 4 & 32 & -24 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 4 \\ 3 \\ 4 \end{bmatrix}$

2c.) $\begin{bmatrix} 6 & 24 & -2 & -4 & -6 \\ 3 & 12 & 0 & 6 & -9 \\ 0 & 0 & 4 & 32 & -24 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

$$\left[\begin{array}{ccccc|cc} 6 & 24 & -2 & -4 & -6 & 4 & 1 \\ 3 & 12 & 0 & 6 & -9 & 3 & 0 \\ 0 & 0 & 4 & 32 & -24 & 4 & 1 \end{array} \right]$$

$\downarrow R_1 \leftrightarrow R_2$

$$\left[\begin{array}{ccccc|cc} 3 & 12 & 0 & 6 & -9 & 3 & 0 \\ 6 & 24 & -2 & -4 & -6 & 4 & 1 \\ 0 & 0 & 4 & 32 & -24 & 4 & 1 \end{array} \right]$$

$\downarrow R_2 - 2R_1$

$$\left[\begin{array}{ccccc|cc} 3 & 12 & 0 & 6 & -9 & 3 & 0 \\ 0 & 0 & -2 & -16 & 12 & -2 & 1 \\ 0 & 0 & 4 & 32 & -24 & 4 & 1 \end{array} \right]$$

$\downarrow R_3 + 2R_2, R_1/3, R_2/(-2)$

$$\begin{array}{c} x_1 \quad x_3 \quad x_2 \quad x_4 \quad x_5 \\ \left[\begin{array}{ccccc|cc} 1 & 4 & 0 & 2 & -3 & 1 & 0 \\ 0 & 0 & 1 & 8 & -6 & 1 & -1/2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 3 \end{array} \right] \end{array}$$

\rightarrow 2c) no sol'n

$$x_1 = -4x_2 - 2x_4 + 3x_5 + 1$$

$$x_2 = x_2 + 0x_4 + 0x_5 + 0$$

$$x_3 = -8x_4 + 6x_5 + 1$$

$$x_4 = 0x_2 + 1x_4 + 0x_5 + 0$$

$$x_5 = 0x_2 + 0x_5 + 1x_5 + 0$$

Part 2: Multiple Choice (T/F are worth 4 points each, while the remaining multiple choice problems are worth 6 points each).

Problem 1. Every linear system with free variables has infinitely many solutions.

- A. True
- B. False

or no sol'n

Problem 2. A system of linear equations can have exactly 3 solutions.

- A. True
- B. False

0, 1, ∞

Problem 3. If the equation $Ax = b$ is consistent if and only if b is **not** in the set spanned by the columns of A .

- A. True
- B. False

in

Problem 4. Any linear combination of vectors can always be written in the form Ax for a suitable matrix A and vector x .

- A. True
 - B. False
-

Problem 5. Determine if the matrix

$$\begin{bmatrix} -2 & -4 & 2 & -2 & 0 \\ 0 & 0 & -3 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

is in echelon form, reduced echelon form, or neither. Choose the *most appropriate* answer.

- A. echelon form
- B. reduced echelon form
- C. neither

Problem 6. Which of the following best describes the span of the 3 vectors below?

Let $A = \begin{bmatrix} -5 \\ -4 \\ -3 \end{bmatrix}$, $B = \begin{bmatrix} -50 \\ -40 \\ -30 \end{bmatrix}$, and $C = \begin{bmatrix} 10 \\ 11 \\ 4 \end{bmatrix}$.

line

plane

- A. 0-dimensional point in \mathbb{R}^3
- B. 1-dimensional line in \mathbb{R}^3
- C. 2-dimensional plane in \mathbb{R}^3
- D. \mathbb{R}^3
- E. None of the above.

Problem 7. Assume $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ does not span \mathbb{R}^3 . Select the best statement.

- A. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ spans \mathbb{R}^3 unless \mathbf{u}_4 is the zero vector.
- B. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ spans \mathbb{R}^3 unless \mathbf{u}_4 is a scalar multiple of another vector in the set.
- C. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ spans \mathbb{R}^3 unless \mathbf{u}_4 is a linear combination of the other vectors in the set.
- D. There is no easy way to determine if $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ spans \mathbb{R}^3 .
- E. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ always spans \mathbb{R}^3 .
- F. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ never spans \mathbb{R}^3 .
- G. none of the above

Eg $\{\vec{u}_1, 2\vec{u}_1, 3\vec{u}_1, 4\vec{u}_1\}$
 or $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$

Problem 8. What conditions on a matrix A insures that $A\mathbf{x} = \mathbf{b}$ has a solution for all \mathbf{b} in \mathbb{R}^n ?
 Select the best statement. (The best condition should work with any positive integer n .)

- A. The equation will have a solution for all \mathbf{b} in \mathbb{R}^n as long as the columns of A do not include the zero column. ~~X~~
- B. The equation will have a solution for all \mathbf{b} in \mathbb{R}^n as long as no column of A is a scalar multiple of another column. ~~X~~
- C. The equation will have a solution for all \mathbf{b} in \mathbb{R}^n as long as no column of A is a linear combination of the other columns of A . ~~X~~
- D. The equation will have a solution for all \mathbf{b} in \mathbb{R}^n as long as the columns of A are linear independent. ~~X~~
- E. The equation will have a solution for all \mathbf{b} in \mathbb{R}^n as long as the columns of A span \mathbb{R}^n .
- F. There is no easy test to determine if the equation will have a solution for all \mathbf{b} in \mathbb{R}^n .
- G. none of the above

no f.v.

Problem 9. Determine which of the following sets of vectors are linearly independent (choose exactly one).

• A. $\begin{bmatrix} -9 \\ 0 \\ 6 \end{bmatrix}, \begin{bmatrix} 4 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ -7 \\ -8 \end{bmatrix}, \begin{bmatrix} -5 \\ 4 \\ 1 \end{bmatrix}$ $4 > 3 \times$

• B. $\begin{bmatrix} 10 \\ -2 \end{bmatrix}, \begin{bmatrix} 5 \\ -1 \end{bmatrix}$ mult

• C. $\begin{bmatrix} 8 \\ -5 \end{bmatrix}, \begin{bmatrix} 4 \\ -2 \end{bmatrix}, \begin{bmatrix} 7 \\ 3 \end{bmatrix}$ $3 > 2$

• D. $\begin{bmatrix} 5 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} -8 \\ 0 \\ -9 \end{bmatrix}, \begin{bmatrix} 6 \\ 0 \\ -3 \end{bmatrix}$ ~~mult~~

• E. $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 7 \\ 2 \\ -3 \end{bmatrix}, \begin{bmatrix} 4 \\ 5 \\ 8 \end{bmatrix}$

• F. $\begin{bmatrix} -4 \\ -7 \\ 8 \end{bmatrix}, \begin{bmatrix} -5 \\ 4 \\ -2 \end{bmatrix}, \begin{bmatrix} 1 \\ -11 \\ 11 \end{bmatrix}$

• G. $\begin{bmatrix} 7 \\ 3 \end{bmatrix}, \begin{bmatrix} -7 \\ -3 \end{bmatrix}$ mult

at most 2 pivots
 \Rightarrow f.v since $3 > 2$

By eliminating others

Problem 10. Give a geometric description of the following system of equations

$$\begin{cases} -5x - 5y = 3 \\ 10x - y = -6 \\ 5x + 10y = -1 \end{cases}$$

$$\left[\begin{array}{cc|c} -5 & -5 & 3 \\ 10 & -1 & -6 \\ 5 & 10 & -1 \end{array} \right] \rightarrow \left[\begin{array}{cc|c} 5 & 5 & -3 \\ 0 & -11 & 0 \\ 0 & 5 & 2 \end{array} \right]$$

- A. Three identical lines
- B. A set of parallel lines
- C. Three lines intersecting at a single point
- **D. Three non-parallel lines with no common intersection**
- E. Three identical planes
- F. Three planes with no common intersection
- G. Three planes intersecting at a point
- H. Three planes intersecting in a line

3 pivot columns

\Downarrow
no soln

3 lines w/ different slopes

Problem 11. Determine which of the following sets of vectors span \mathbb{R}^3 (choose exactly one).

• A. $\begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 6 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -3 \\ 2 \\ 2 \end{bmatrix}$

2 pivots

Need 3 pivots

• B. $\begin{bmatrix} 2 \\ -3 \\ -1 \end{bmatrix}, \begin{bmatrix} -5 \\ -7 \\ 7 \end{bmatrix}, \begin{bmatrix} 6 \\ 4 \\ -8 \end{bmatrix}, \begin{bmatrix} 6 \\ 4 \\ -8 \end{bmatrix}$

by elimination

• C. $\begin{bmatrix} -1 \\ -3 \\ 3 \end{bmatrix}, \begin{bmatrix} -3 \\ -9 \\ 8 \end{bmatrix}$

2 pivots x

• D. $\begin{bmatrix} 3 \\ 3 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -2 \\ -2 \\ -2 \end{bmatrix}$

1 pivot x

• E. $\begin{bmatrix} -1 \\ -8 \end{bmatrix}, \begin{bmatrix} 5 \\ -1 \end{bmatrix}$

2 x

• F. $\begin{bmatrix} 8 \\ -5 \end{bmatrix}, \begin{bmatrix} 4 \\ -2 \end{bmatrix}, \begin{bmatrix} 7 \\ 3 \end{bmatrix}$

2 x

• G. $\begin{bmatrix} 0 \\ 2 \\ 5 \end{bmatrix}, \begin{bmatrix} 0 \\ -9 \\ 9 \end{bmatrix}, \begin{bmatrix} 0 \\ -3 \\ 4 \end{bmatrix}$

2 x

• H. $\begin{bmatrix} 7 \\ 3 \end{bmatrix}, \begin{bmatrix} -7 \\ -3 \end{bmatrix}$

1 x

need vectors in \mathbb{R}^3 (so each vector has 3 coordinates)