Math 2550 Matrix Algebra Exam 1 **Form C**
Sept 24, 2014

[12] 1.) The following matrices are all row equivalent. Use that information to fill in the 9 blanks below:

$$\begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} -8 & -6 & -4 & -38 \\ 12 & 5 & 11 & 43 \\ 8 & 10 & 0 & 50 \\ -12 & -1 & -17 & -27 \end{bmatrix} \sim \begin{bmatrix} 4 & 3 & 2 & 19 \\ 0 & -4 & 5 & -14 \\ 0 & 0 & -1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \sim$$

Determine the following linear combination relationships below (i.e., fill in the 9 blanks below:

$$\underline{\ 5\ } \begin{bmatrix} -8 \\ 12 \\ 8 \\ -12 \end{bmatrix} + \underline{\ 1\ } \begin{bmatrix} -6 \\ 5 \\ 10 \\ -1 \end{bmatrix} + \underline{\ -2\ } \begin{bmatrix} -4 \\ 11 \\ 0 \\ -17 \end{bmatrix} = \begin{bmatrix} -38 \\ 43 \\ 50 \\ -27 \end{bmatrix}$$

$$\underline{\ 5\ } \begin{bmatrix} 4 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \underline{\ 1\ } \begin{bmatrix} 3 \\ -4 \\ 0 \\ 0 \end{bmatrix} + \underline{\ -2\ } \begin{bmatrix} 2 \\ 5 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 19 \\ -14 \\ 2 \\ 0 \end{bmatrix}$$

$$\underline{\ 5\ } \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \underline{\ 1\ } \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} + \underline{\ -2\ } \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 5 \\ 1 \\ -2 \\ 0 \end{bmatrix}$$

[30] 2.) Solve the following systems of equations. Write your answer in parametric vector format.

SHOW YOUR WORK on NEXT PAGE.

2a.) $\begin{bmatrix} 6 & 24 & -2 & -4 & -6 \\ 3 & 12 & 0 & 6 & -9 \\ 0 & 0 & 4 & 32 & -24 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

Answer: $\begin{bmatrix} -4 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} x_2 + \begin{bmatrix} -2 \\ 0 \\ -8 \\ 1 \\ 0 \end{bmatrix} x_4 + \begin{bmatrix} 3 \\ 0 \\ 6 \\ 0 \\ 1 \end{bmatrix} x_5$

2b.) $\begin{bmatrix} 6 & 24 & -2 & -4 & -6 \\ 3 & 12 & 0 & 6 & -9 \\ 0 & 0 & 4 & 32 & -24 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 4 \\ 3 \\ 4 \end{bmatrix}$

Answer: $\begin{bmatrix} -4 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} x_2 + \begin{bmatrix} -2 \\ 0 \\ -8 \\ 1 \\ 0 \end{bmatrix} x_4 + \begin{bmatrix} 3 \\ 0 \\ 6 \\ 0 \\ 1 \end{bmatrix} x_5 + \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$

2c.) $\begin{bmatrix} 6 & 24 & -2 & -4 & -6 \\ 3 & 12 & 0 & 6 & -9 \\ 0 & 0 & 4 & 32 & -24 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

Answer: no solution

Scratch work for 2a.) $\begin{bmatrix} 6 & 24 & -2 & -4 & -6 \\ 3 & 12 & 0 & 6 & -9 \\ 0 & 0 & 4 & 32 & -24 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

2b.) $\begin{bmatrix} 6 & 24 & -2 & -4 & -6 \\ 3 & 12 & 0 & 6 & -9 \\ 0 & 0 & 4 & 32 & -24 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 4 \\ 3 \\ 4 \end{bmatrix}$

2c.) $\begin{bmatrix} 6 & 24 & -2 & -4 & -6 \\ 3 & 12 & 0 & 6 & -9 \\ 0 & 0 & 4 & 32 & -24 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

$$\left[\begin{array}{ccccc|cc} 6 & 24 & -2 & -4 & -6 & 4 & 1 \\ 3 & 12 & 0 & 6 & -9 & 3 & 0 \\ 0 & 0 & 4 & 32 & -24 & 4 & 1 \end{array}\right]$$

$$\downarrow R_1 \leftrightarrow R_2$$

$$\left[\begin{array}{ccccc|cc} 3 & 12 & 0 & 6 & -9 & 3 & 0 \\ 6 & 24 & -2 & -4 & -6 & 4 & 1 \\ 0 & 0 & 4 & 32 & -24 & 4 & 1 \end{array}\right]$$

$$\downarrow R_2 - 2R_1$$

$$\left[\begin{array}{ccccc|cc} 3 & 12 & 0 & 6 & -9 & 3 & 0 \\ 0 & 0 & -2 & -16 & 12 & -2 & 1 \\ 0 & 0 & 4 & 32 & -24 & 4 & 1 \end{array}\right]$$

$$\downarrow R_3 + 2R_2, \quad R_1/3, \quad R_2/(-2)$$

$$\begin{array}{c} \begin{array}{ccccc} x_1 & X_3 & x_2 & x_4 & x_5 \end{array} \\ \left[\begin{array}{ccccc|cc} 1 & 4 & 0 & 2 & -3 & 1 & 0 \\ 0 & 0 & 1 & 8 & -6 & 1 & -1/2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 3 \end{array}\right] \end{array}$$

$\hookrightarrow$ 2c) no sol'n

$$x_1 = -4x_2 - 2x_4 + 3x_5 + 1$$
$$X_2 = X_2 + 0x_4 + 0x_5 + 0$$
$$X_3 = \quad - 8x_4 + 6x_5 + 1$$
$$X_4 = 0X_2 + 1X_4 + 0x_5 + 0$$
$$X_5 = 0X_2 + 0x_5 + 1X_5 + 0$$

Part 2: Multiple Choice (T/F are worth 4 points each, while the remaining multiple choice problems are worh 6 points each).

Problem 1. A vector b is a linear combination of the columns of a matrix A if and only if the equation $Ax = b$ has at least one solution.

- A. True
- B. False

Problem 2. A system of linear equations has an infinite number of solutions if and only if its associated augmented matrix has a column corresponding to a free variable

$\Rightarrow$ ✓

- A. True
- B. False

∞ or none $\Leftarrow$

Problem 3. If the equation $Ax = b$ is **consistent** if and only if b is in the set spanned by the columns of A.

- A. True
- B. False

Problem 4. If A is an $m \times n$ matrix whose columns span $\mathbb{R}^m$, then the equation $Ax = b$ is **consistent** for all b in $\mathbb{R}^m$.

- A. True
- B. False

Problem 5. Which of the following best describes the span of the 3 vectors below?

line

Let $A = \begin{bmatrix} -5 \\ -4 \\ -3 \end{bmatrix}$, $B = \begin{bmatrix} -50 \\ -40 \\ -30 \end{bmatrix}$, and $C = \begin{bmatrix} 10 \\ 11 \\ 4 \end{bmatrix}$.

plane

- A. 0-dimensional point in R^3
- B. ~~1-dimensional line in R^3~~
- C. 2-dimensional plane in R^3
- D. R^3
- E. None of the above.

Problem 6. Determine if the matrix

$$\begin{bmatrix} -2 & -4 & 2 & -2 & 0 \\ 0 & 0 & -3 & 2 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

is in echelon form, reduced echelon form, or neither. Choose the *most appropriate* answer.

- A. echelon form
- B. reduced echelon form
- C. neither

Problem 7. What conditions on a matrix A insures that $A\mathbf{x} = \mathbf{b}$ has a solution for all $\mathbf{b}$ in $\mathbb{R}^n$? Select the best statement. (The best condition should work with any positive integer n.)

- A. The equation will have a solution for all $\mathbf{b}$ in $\mathbb{R}^n$ as long as the columns of A do not include the zero column.
- B. The equation will have a solution for all $\mathbf{b}$ in $\mathbb{R}^n$ as long as no column of A is a scalar multiple of another column.
- C. The equation will have a solution for all $\mathbf{b}$ in $\mathbb{R}^n$ as long as no column of A is a linear combination of the other columns of A.
- D. The equation will have a solution for all $\mathbf{b}$ in $\mathbb{R}^n$ as long as the columns of A are linear independent.
- E. The equation will have a solution for all $\mathbf{b}$ in $\mathbb{R}^n$ as long as the columns of A span $\mathbb{R}^n$.
- F. There is no easy test to determine if the equation will have a solution for all $\mathbf{b}$ in $\mathbb{R}^n$.
- G. none of the above

Problem 8. Give a geometric description of the following system of equations

$$\begin{aligned} -5x - 5y &= 2 \\ 10x + y &= -4 \\ 5x + 10y &= -2 \end{aligned}$$

$$\left[\begin{array}{cc|c} 5 & 5 & -2 \\ 0 & 9 & -8 \\ 0 & 5 & 6 \end{array}\right]$$

- A. Three identical lines
- B. A set of parallel lines
- C. Three lines intersecting at a single point
- D. Three non-parallel lines with no common intersection
- E. Three identical planes
- F. Three planes with no common intersection
- G. Three planes intersecting at a point
- H. Three planes intersecting in a line

Need 3 pivots

Problem 9. Determine which of the following sets of vectors span $\mathbb{R}^3$ **(choose exactly one)**.

need vectors in $\mathbb{R}^3$ (so each vector has 3 coordinates

- A. $\begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix}, \cancel{\begin{bmatrix} 6 \\ 0 \\ 0 \end{bmatrix}}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -3 \\ 2 \\ 2 \end{bmatrix}$ 2 pivots
- B. $\begin{bmatrix} 2 \\ -3 \\ -1 \end{bmatrix}, \begin{bmatrix} -5 \\ -7 \\ 7 \end{bmatrix}, \begin{bmatrix} 6 \\ 4 \\ -8 \end{bmatrix}, \cancel{\begin{bmatrix} 6 \\ 4 \\ -8 \end{bmatrix}}$ by elimination
- C. $\begin{bmatrix} -1 \\ -3 \\ 3 \end{bmatrix}, \begin{bmatrix} -3 \\ -9 \\ 8 \end{bmatrix}$ 2 pivots
- D. $\begin{bmatrix} 3 \\ 3 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -2 \\ -2 \\ -2 \end{bmatrix}$ 1 pivot
- E. $\begin{bmatrix} -1 \\ -8 \end{bmatrix}, \begin{bmatrix} 5 \\ -1 \end{bmatrix}$ 2 pivots
- F. $\begin{bmatrix} 8 \\ -5 \end{bmatrix}, \begin{bmatrix} 4 \\ -2 \end{bmatrix}, \begin{bmatrix} 7 \\ 3 \end{bmatrix}$ 2
- G. $\begin{bmatrix} 0 \\ 2 \\ 5 \end{bmatrix}, \begin{bmatrix} 0 \\ -9 \\ 9 \end{bmatrix}, \begin{bmatrix} 0 \\ -3 \\ 4 \end{bmatrix}$ 2
- H. $\begin{bmatrix} 7 \\ 3 \end{bmatrix}, \begin{bmatrix} -7 \\ -3 \end{bmatrix}$ 1

no f.v.

Problem 10. Determine which of the following sets of vectors are linearly independent **(choose exactly one)**.

- A. $\begin{bmatrix} -9 \\ 0 \\ 6 \end{bmatrix}, \begin{bmatrix} 4 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ -7 \\ -8 \end{bmatrix}, \begin{bmatrix} -5 \\ 4 \\ 1 \end{bmatrix}$ × $4 > 3$
- B. $\begin{bmatrix} 10 \\ -2 \end{bmatrix}, \begin{bmatrix} 5 \\ -1 \end{bmatrix}$ × mult
- C. $\begin{bmatrix} 8 \\ -5 \end{bmatrix}, \begin{bmatrix} 4 \\ -2 \end{bmatrix}, \begin{bmatrix} 7 \\ 3 \end{bmatrix}$ × $3 > 2$
- D. $\begin{bmatrix} 5 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} -8 \\ 0 \\ -9 \end{bmatrix}, \begin{bmatrix} 6 \\ 0 \\ -3 \end{bmatrix}$ ×
- E. $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 7 \\ 2 \\ -3 \end{bmatrix}, \begin{bmatrix} 4 \\ 5 \\ 8 \end{bmatrix}$ ×

f.v.

at most 2 pivots $\Rightarrow$ f.v since $3 > 2$

- F. $\begin{bmatrix} -4 \\ -7 \\ 8 \end{bmatrix}, \begin{bmatrix} -5 \\ 4 \\ -2 \end{bmatrix}, \begin{bmatrix} 1 \\ -11 \\ 11 \end{bmatrix}$ $\Leftarrow$ By elimination
- G. $\begin{bmatrix} 7 \\ 3 \end{bmatrix}, \begin{bmatrix} -7 \\ -3 \end{bmatrix}$ × mult

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Problem 11. Assume $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ does **not** span $\mathbb{R}^3$. Select the best statement.

- A. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ spans $\mathbb{R}^3$ unless $\mathbf{u}_4$ is the zero vector.
- B. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ spans $\mathbb{R}^3$ unless $\mathbf{u}_4$ is a scalar multiple of another vector in the set.
- C. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ spans $\mathbb{R}^3$ unless $\mathbf{u}_4$ is a linear combination of the other vectors in the set.
- D. There is no easy way to determine if $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ spans $\mathbb{R}^3$.
- E. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ always spans $\mathbb{R}^3$.
- F. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ never spans $\mathbb{R}^3$.
- G. none of the above

E.g: $\{\vec{u}_1, 2\vec{u}_1, 3\vec{u}_1, 4\vec{u}_1\}$

or $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$