[12] 1.) The following matrices are all row equivalent. Use that information to fill in the 9 blanks below:

$$\begin{bmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} -6 & 4 & -10 & 32 \\ 9 & -8 & 11 & -28 \\ 6 & -6 & 0 & 6 \\ 6 & -8 & -4 & 26 \end{bmatrix} \sim \begin{bmatrix} 3 & -2 & 5 & -16 \\ 0 & -2 & -4 & 20 \\ 0 & 0 & -3 & 9 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Determine the following linear combination relationships below (i.e., fill in the 9 blanks below):

$$\begin{array}{c|c}
-3 & \begin{bmatrix} -6\\9\\6\\6 \end{bmatrix} + \frac{-4}{6} & \begin{bmatrix} 4\\-8\\-6\\-8 \end{bmatrix} + \frac{-3}{6} & \begin{bmatrix} -10\\11\\0\\-4 \end{bmatrix} = \begin{bmatrix} 32\\-28\\6\\26 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -3 \\ -4 \\ -3 \\ 0 \end{bmatrix}$$

[30] 2.) Solve the following systems of equations. Write your answer in parametric vector format. **SHOW YOUR WORK on NEXT PAGE**.

2a.)
$$\begin{bmatrix} 6 & 24 & -2 & -4 & -6 \\ 3 & 12 & 0 & 6 & -9 \\ 0 & 0 & 4 & 32 & -24 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Answer:
$$\begin{bmatrix} -4 \\ 8 \\ 8 \end{bmatrix}$$

2b.) $\begin{bmatrix} 6 & 24 & -2 & -4 & -6 \\ 3 & 12 & 0 & 6 & -9 \\ 0 & 0 & 4 & 32 & -24 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 4 \\ 3 \\ 4 \end{bmatrix}$

Answer:

2c.)
$$\begin{bmatrix} 6 & 24 & -2 & -4 & -6 \\ 3 & 12 & 0 & 6 & -9 \\ 0 & 0 & 4 & 32 & -24 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

Scratch work for 2a.)
$$\begin{bmatrix} 6 & 24 & -2 & -4 & -6 \ 3 & 12 & 0 & 6 & -9 \ 0 & 0 & 4 & 32 & -24 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 0 \ 0 \ 0 \end{bmatrix}$$
2b.)
$$\begin{bmatrix} 6 & 24 & -2 & -4 & -6 \ 3 & 12 & 0 & 6 & -9 \ 0 & 0 & 4 & 32 & -24 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 4 \ 3 \ 4 \end{bmatrix}$$

$$\begin{bmatrix} 6 & 24 & -2 & -4 & -6 \ 3 & 12 & 0 & 6 & -9 \ 0 & 0 & 4 & 32 & -24 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 4 \ 3 \ 4 \end{bmatrix}$$

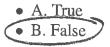
$$\begin{bmatrix} 6 & 24 & -2 & -4 & -6 \ 3 & 12 & 0 & 6 & -9 \ 4 & 32 & -24 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 4 \ 3 \ 4 \end{bmatrix}$$

$$\begin{bmatrix} 7 & 12 & 0 & 6 & -9 \ 4 & 32 & -24 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 9 & 12 \ 4 & 11 \end{bmatrix}$$

$$\begin{bmatrix} 7 & 12 & 0 & 6 & -9 \ 4 & 32 & -24 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 9 & 13 & 0 \ 4 & 32 & -24 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 9 & 13 & 0 \ 4 & 32 & -24 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 9 & 13 & 0 \ 4 & 32 & -24 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 9 & 13 & 0 \ 4 & 32 & -24 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 9 & 13 & 0 \ 4 & 32 & -24 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 9 & 13 & 0 \ 14 & 11 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 9 &$$

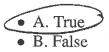
Part 2: Multiple Choice (T/F are worth 4 points each, while the remaining multiple choice problems are worh 6 points each).

Problem 1. If a linear system has five equations and nine variables, then it must have infinitely many solutions.

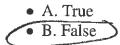


or no sol'n

Problem 2. A vector b is a linear combination of the columns of a matrix A if and only if the equation Ax = b has at least one solution.

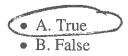


Problem 3. If the equation Ax = b is **inconsistent** if and only if b is in the set spanned by the columns of A.



NOT

Problem 4. If the columns of an $m \times n$ matrix, A span \mathbb{R}^m , then the equation, Ax = b is **consistent** for each b in \mathbb{R}^m .



Problem 5. What conditions on a matrix A insures that $A\mathbf{x} = \mathbf{b}$ has a solution for all \mathbf{b} in \mathbb{R}^n ? Select the best statement. (The best condition should work with any positive integer n.)

- A. The equation will have a solution for all **b** in \mathbb{R}^n as long as the columns of A do not include the zero column.
- B. The equation will have a solution for all **b** in \mathbb{R}^n as long as no column of A is a scalar multiple of another column.
- C. The equation will have a solution for all **b** in \mathbb{R}^n as long as no column of A is a linear combination of the other columns of A.
- D. The equation will have a solution for all b in \mathbb{R}^n as long as the columns of A are linear independent.
- F The equation will have a solution for all **b** in \mathbb{R}^n as long as the columns of A span \mathbb{R}^n .
 - F. There is no easy test to determine if the equation will have a solution for all b in \mathbb{R}^n .
 - G. none of the above

Problem 6. Determine if the matrix

$$\begin{bmatrix}
1 & 3 & 0 & -9 & -4 \\
0 & 0 & 1 & 6 & 7 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix}$$

is in echelon form, reduced echelon form, or neither. Choose the most appropriate answer.

• A. echelon form

B. reduced echelon form

• C. neither

Problem 7. Assume $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ spans \mathbb{R}^3 . Select the best statement.

- A. $\{u_1, u_2, u_3, u_4\}$ spans \mathbb{R}^3 unless u_4 is the zero vector.
- B. $\{u_1, u_2, u_3, u_4\}$ spans \mathbb{R}^3 unless u_4 is a scalar multiple of another vector in the set.
- C. $\{u_1, u_2, u_3, u_4\}$ spans \mathbb{R}^3 unless u_4 is a linear combination of the other vectors in the set.
- D. There is no easy way to determine if $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ spans \mathbb{R}^3 .

E. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ always spans \mathbb{R}^3 . F. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ never spans \mathbb{R}^3 .

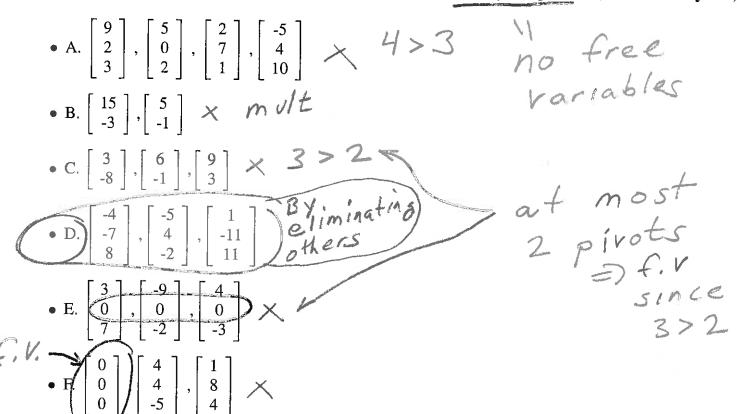
• G. none of the above

span [4, 42, 43] = Span [4, 42, 43, 44]

Problem 8. Give a geometric description of the following system of equations

- $\begin{bmatrix} 3 & 5 & 7 \\ -7 & 8 & -4 \end{bmatrix}$
- A. Three identical lines
- B. A set of parallel lines
- C. Three lines intersecting at a single point
- D. Three non-parallel lines with no common intersection
 - E. Three identical planes
 - F. Three planes with no common intersection
 - G. Three planes intersecting at a point
 - H. Three planes intersecting in a line

Problem 9. Determine which of the following sets of vectors are linearly independent (choose exactly one).



Problem 10. Determine which of the following sets of vectors span \mathbb{R}^3 (choose exactly one). 2 need vectors in R3 (so each

rector has
3 coordinates)

Need 3 pivots

• A.
$$\begin{bmatrix} 0 \\ 2 \\ 5 \end{bmatrix}$$
, $\begin{bmatrix} 0 \\ -9 \\ 9 \end{bmatrix}$, $\begin{bmatrix} 0 \\ -3 \\ 4 \end{bmatrix}$ \times 2 prof 5
• B. $\begin{bmatrix} 3 \\ -5 \end{bmatrix}$, $\begin{bmatrix} 9 \\ -2 \end{bmatrix}$, $\begin{bmatrix} 7 \\ 6 \end{bmatrix}$ \star 2 prof 5

• B.
$$\begin{bmatrix} 3 \\ -5 \end{bmatrix}$$
, $\begin{bmatrix} 9 \\ -2 \end{bmatrix}$, $\begin{bmatrix} 7 \\ 6 \end{bmatrix}$ \swarrow 2 pivots

• C.
$$\begin{bmatrix} -1 \\ -5 \\ 3 \end{bmatrix}$$
, $\begin{bmatrix} -3 \\ -9 \\ 2 \end{bmatrix}$ \swarrow 2

• D.
$$\begin{bmatrix} 4 \\ 4 \\ 4 \end{bmatrix}$$
, $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, $\begin{bmatrix} -2 \\ -2 \\ -2 \end{bmatrix}$

• E.
$$\begin{bmatrix} -1 \\ -8 \end{bmatrix}$$
, $\begin{bmatrix} 5 \\ -1 \end{bmatrix}$

• E.
$$\begin{bmatrix} -1 \\ -8 \end{bmatrix}$$
, $\begin{bmatrix} 3 \\ -1 \end{bmatrix}$

• F. $\begin{bmatrix} 2 \\ -3 \\ -1 \end{bmatrix}$, $\begin{bmatrix} -5 \\ -7 \\ 7 \end{bmatrix}$, $\begin{bmatrix} 6 \\ 4 \\ -8 \end{bmatrix}$ $\begin{bmatrix} 6 \\ 4 \\ -8 \end{bmatrix}$

• G.
$$\begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix}$$
, $\begin{bmatrix} 6 \\ 0 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, $\begin{bmatrix} -3 \\ 2 \\ 2 \end{bmatrix}$ \times 2 pluots

• H.
$$\begin{bmatrix} 7 \\ 3 \end{bmatrix}$$
, $\begin{bmatrix} -7 \\ -3 \end{bmatrix}$

Problem 11.

Which of the following best describes the span of the 3 vectors below?

Let
$$A = \begin{bmatrix} -5 \\ -4 \\ -3 \end{bmatrix}$$
, $B = \begin{bmatrix} -50 \\ 40 \\ -30 \end{bmatrix}$, and $C = \begin{bmatrix} 10 \\ 11 \\ 4 \end{bmatrix}$.

- A. 0-dimensional point in R^3
- B. 1-dimensional line in R³
- C. 2-dimensional plane in R^3 $D_{\cdot}R^3$
- E. None of the above.