[12] 1.) The following matrices are all row equivalent. Use that information to fill in the 9 blanks below:

$$\begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 8 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 4 & -2 & 1 & -11 \\ -8 & 7 & -4 & 52 \\ 16 & -2 & 1 & 13 \\ 20 & -4 & 2 & 2 \end{bmatrix} \sim \begin{bmatrix} 4 & -2 & 1 & -11 \\ 0 & 3 & -2 & 30 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Determine the following linear combination relationships below (i.e., fill in the 9 blanks below:

[30] 2.) Solve the following systems of equations. Write your answer in parametric vector format. **SHOW YOUR WORK on NEXT PAGE**.

2a.)
$$\begin{bmatrix} 6 & 24 & -2 & -4 & -6 \\ 3 & 12 & 0 & 6 & -9 \\ 0 & 0 & 4 & 32 & -24 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Answer:
$$\begin{bmatrix} -2 \\ -2 \\ 0 \\ -8 \end{bmatrix} \times \begin{bmatrix} -2 \\ 0 \\ -8 \end{bmatrix} \times \begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix} \times \begin{bmatrix} 3 \\ 0 \\ 0$$

2b.) $\begin{bmatrix} 6 & 24 & -2 & -4 & -6 \\ 3 & 12 & 0 & 6 & -9 \\ 0 & 0 & 4 & 32 & -24 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 4 \\ 3 \\ 4 \end{bmatrix}$

2c.) $\begin{bmatrix} 6 & 24 & -2 & -4 & -6 \\ 3 & 12 & 0 & 6 & -9 \\ 0 & 0 & 4 & 32 & -24 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

Answer:_____

Scratch work for 2a.)
$$\begin{bmatrix} 6 & 24 & -2 & -4 & -6 \ 3 & 12 & 0 & 6 & -9 \ 0 & 0 & 4 & 32 & -24 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 0 \ 0 \ 0 \end{bmatrix}$$
2b.)
$$\begin{bmatrix} 6 & 24 & -2 & -4 & -6 \ 3 & 12 & 0 & 6 & -9 \ 0 & 0 & 4 & 32 & -24 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 4 \ 3 \ 4 \end{bmatrix}$$

$$\begin{bmatrix} 6 & 24 & -2 & -4 & -6 \ 3 & 12 & 0 & 6 & -9 \ 0 & 0 & 4 & 32 & -24 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 4 \ 3 \ 4 \end{bmatrix}$$

$$\begin{bmatrix} 6 & 24 & -2 & -4 & -6 \ 3 & 12 & 0 & 6 & -9 \ 4 & 32 & -24 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 4 \ 3 \ 4 \end{bmatrix}$$

$$\begin{bmatrix} 6 & 24 & -2 & -4 & -6 \ 3 & 4 & 32 & -24 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 4 \ 3 \ 4 \end{bmatrix}$$

$$\begin{bmatrix} 6 & 24 & -2 & -4 & -6 \ 3 & 4 & 32 & -24 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 1 \ 2 \ 3 \ 4 \end{bmatrix}$$

$$\begin{bmatrix} 6 & 24 & -2 & -4 & -6 \ 3 & 4 & 32 & -24 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 1 \ 2 \ 3 \ 4 \end{bmatrix}$$

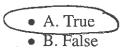
$$\begin{bmatrix} 6 & 24 & -2 & -4 & -6 \ 3 & 4 & 32 & -24 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 1 \ 2 \ 3 \ 4 \end{bmatrix}$$

$$\begin{bmatrix} 6 & 24 & -2 & -4 & -6 \ 3 & 4 & 32 & -24 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 1 \ 2 \ 3 \ 4 \end{bmatrix}$$

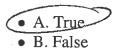
$$2c.) \begin{bmatrix} 6 & 24 & -2 & -4 & -6 \\ 3 & 12 & 0 & 6 & -9 \\ 0 & 0 & 4 & 32 & -24 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

Part 2: Multiple Choice (T/F are worth 4 points each, while the remaining multiple choice problems are worh 6 points each).

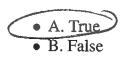
Problem 1. Any linear combination of vectors can always be written in the form Ax for a suitable matrix A and vector x.



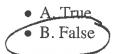
Problem 2. A system of linear equations has no solution if and only if the last column of its augmented matrix corresponds to a pivot column.



Problem 3. If the equation Ax = b is **consistent** if and only if b is in the set spanned by the columns of A.



Problem 4. If A is an $m \times n$ matrix whose columns span \mathbb{R}^m , then the equation Ax = b is **beconsistent** for some b in \mathbb{R}^m .



Problem 5. Which of the following best describes the span of the 3 vectors below?

Let
$$A = \begin{bmatrix} -5 \\ -4 \\ -3 \end{bmatrix}$$
, $B = \begin{bmatrix} -50 \\ -40 \\ -30 \end{bmatrix}$, and $C = \begin{bmatrix} 10 \\ 11 \\ 4 \end{bmatrix}$

- A. 0-dimensional point in R^3
- B. 1-dimensional line in R^3
- C. 2-dimensional plane in R^3
- D. R³
- E. None of the above.

Problem 6. Determine if the matrix

$$\begin{bmatrix}
1 & -1 & 0 & -8 & 0 \\
0 & 0 & 1 & -9 & 0 \\
0 & 0 & 0 & 0 & 1
\end{bmatrix}$$

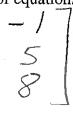
is in echelon form, reduced echelon form, or neither. Choose the most appropriate answer.

- A. echelon form
- B. reduced echelon form
- C. neither

Problem 7. Assume $\{u_1, u_2, u_3\}$ spans \mathbb{R}^3 . Select the best statement.

- A. $\{u_1, u_2, u_3, u_4\}$ spans \mathbb{R}^3 unless u_4 is the zero vector.
- B. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ spans \mathbb{R}^3 unless \mathbf{u}_4 is a scalar multiple of another vector in the set.
- C. $\{u_1, u_2, u_3, u_4\}$ spans \mathbb{R}^3 unless u_4 is a linear combination of the other vectors in the set.
- D. There is no easy way to determine if $\{u_1, u_2, u_3, u_4\}$ spans \mathbb{R}^3 .
- E. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ always spans \mathbb{R}^3 .
- F. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ never spans \mathbb{R}^3 .
- G. none of the above

Problem 8. Give a geometric description of the following system of equations



- A. Three identical lines
- B. A set of parallel lines
- C. Three lines intersecting at a single point
- D. Three non-parallel lines with no common intersection
 - E. Three identical planes
 - F. Three planes with no common intersection
 - G. Three planes intersecting at a point
 - H. Three planes intersecting in a line

Need 3 prints

Problem 9. Determine which of the following sets of vectors span \mathbb{R}^3 (choose exactly one)

• A. $\begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 6 \\ 0 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$ 2 pivots

Need vectors in R3

 $\begin{bmatrix} 2 \\ -3 \\ -1 \end{bmatrix}, \begin{bmatrix} -5 \\ -7 \\ 7 \end{bmatrix}, \begin{bmatrix} 6 \\ 4 \\ -8 \end{bmatrix}, \begin{bmatrix} 6 \\ 4 \\ -8 \end{bmatrix}$

vector has 3 condinates

• C. $\begin{bmatrix} -1 \\ -3 \\ 3 \end{bmatrix}$, $\begin{bmatrix} -3 \\ -9 \\ 8 \end{bmatrix}$ \checkmark 2 $p \mid vo^{+} \leq s$

• D. $\begin{bmatrix} 3 \\ 3 \\ 3 \end{bmatrix}$, $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, $\begin{bmatrix} -2 \\ -2 \\ -2 \end{bmatrix}$ \swarrow | plue \leftarrow

• E. $\begin{bmatrix} -1 \\ -8 \end{bmatrix}$, $\begin{bmatrix} 5 \\ -1 \end{bmatrix}$

• F. $\begin{vmatrix} 8 \\ -5 \end{vmatrix}$, $\begin{vmatrix} 4 \\ -2 \end{vmatrix}$, $\begin{vmatrix} 7 \\ 3 \end{vmatrix}$

• G. $\begin{bmatrix} 0 \\ 2 \\ 5 \end{bmatrix}$, $\begin{bmatrix} 0 \\ -9 \\ 9 \end{bmatrix}$, $\begin{bmatrix} 0 \\ -3 \\ 4 \end{bmatrix}$

• H. $\begin{bmatrix} 7 \\ 3 \end{bmatrix}$, $\begin{bmatrix} -7 \\ -3 \end{bmatrix}$

no f. V.

Problem 10. Determine which of the following sets of vectors are linearly independent (choose exactly one).

Problem 11. What conditions on a matrix A insures that $A\mathbf{x} = \mathbf{b}$ has a solution for all \mathbf{b} in \mathbb{R}^n ? Select the best statement. (The best condition should work with any positive integer n.)

- A. The equation will have a solution for all **b** in \mathbb{R}^n as long as the columns of A do not include the zero column.
- B. The equation will have a solution for all b in \mathbb{R}^n as long as no column of A is a scalar multiple of another column.
- C. The equation will have a solution for all **b** in \mathbb{R}^n as long as no column of A is a linear combination of the other columns of A.
- D. The equation will have a solution for all b in \mathbb{R}^n as long as the columns of A are linear independent.
- E. The equation will have a solution for all b in \mathbb{R}^n as long as the columns of A span \mathbb{R}^n .
- F. There is no easy test to determine if the equation will have a solution for all b in \mathbb{R}^n .
- G. none of the above