Math 2550 Matrix Algebra Exam 3 **Form A**
Dec. 4, 2014

[18] 1.) Find the characteristic equation and diagonalize $A = \begin{bmatrix} 20 & 10 \\ 10 & 5 \end{bmatrix}$

Characteristic equation of $A =$ ____________________________________.

$P =$ ______________________ $D =$ ______________________ $P^{-1} =$ ______________________.

[16] 2.) Use Gram-Schmidt to find the QR factorization of $M = \begin{bmatrix} 2 & 0 \\ 1 & 9 \\ 2 & 9 \end{bmatrix}$.

$Q =$ ______________________________ $R =$ ______________________________

Part 2: Multiple Choice (multiple choice problems are worh 6 points each).

Problem 1.
If the characteristic polynomial of $A = (\lambda-4)^8(\lambda+4)^2(\lambda+7)^6$, then the geometric multiplicity of $\lambda = -4$ is

- A. 0
- B. 1
- C. 2
- D. 3
- E. 0 or 1
- F. 0 or 2
- G. 1 or 2
- H. 0, 1, or 2
- I. 0, 1, 2, or 3
- J. none of the above

Problem 2. 0 is an eigenvalue of A if and only if the columns of A are linearly dependent.

- A. True
- B. False

Problem 3. Let $A = \begin{bmatrix} 7 & -1 & 3 \\ 0 & 9 & -6 \\ 0 & 0 & 7 \end{bmatrix}$. Is A = diagonalizable?

- A. yes
- B. no
- C. none of the above

Problem 4. Suppose $A = PDP^{-1}$ where D is a diagonal matrix. Suppose also the d_{ii} are the diagonal entries of D. If $P = [\ \vec{p}_1\ \vec{p}_2\ \vec{p}_3\ ]$ and $d_{11} = d_{22}$, then $3\vec{p}_1 + 4\vec{p}_2$ is an eigenvector of A

- A. True
- B. False

Problem 5. If x is in a subspace W, then $x - \text{proj}_W(x) = 0$.

- A. True
- B. False

Problem 6. If A is symmetric, then A is diagonalizable.

- A. True
- B. False

Problem 7. Suppose $A\begin{bmatrix} 3 \\ -5 \\ 3 \end{bmatrix} = \begin{bmatrix} -12 \\ 20 \\ -12 \end{bmatrix}$. Then an eigenvalue of A is

- A. -4
- B. -3
- C. -2
- D. -1
- E. 0
- F. 1
- G. 2
- H. 3
- I. 4
- J. none of the above

Problem 8. Let $A = \begin{bmatrix} 5 & -1 & 3 \\ 0 & -6 & -2 \\ 0 & 0 & 5 \end{bmatrix}$. Is A = diagonalizable?

- A. yes
- B. no
- C. none of the above

Problem 9. The eigenspace corresponding to a particular eigenvalue of A contains an infinite number of vectors.

- A. True
- B. False

Problem 10.
If the characteristic polynomial of $A = (\lambda+7)^9(\lambda-5)^2(\lambda+3)^4$, then the algebraic multiplicity of $\lambda = 5$ is

- A. 0
- B. 1
- C. 2
- D. 3
- E. 0 or 1
- F. 0 or 2
- G. 1 or 2
- H. 0, 1, or 2
- I. 0, 1, 2, or 3
- J. none of the above

Problem 11. Suppose the orthogonal projection of $\begin{bmatrix} 92 \\ -1 \\ -6 \end{bmatrix}$ onto $\begin{bmatrix} 1 \\ -4 \\ 2 \end{bmatrix}$ is (z_1, z_2, z_3). Then $z_1 =$

- A. -4
- B. -3
- C. -2
- D. -1
- E. 0
- F. 1
- G. 2
- H. 3
- I. 4
- J. none of the above