Math 2550 Matrix Algebra Multiple Choice Exam **Form C**
Oct. 27, 2014

Problem 1. Suppose A is a 9×8 matrix. If rank of $A = 5$, then nullity of $A =$

- A. -4
- B. -3
- C. -2
- D. -1
- E. 0
- F. 1
- G. 2
- H. 3
- I. 4
- J. none of the above

8 5 9 $8-5=3$

Problem 2. Suppose A is a square matrix and $A\vec{x} = \vec{0}$ has an infinite number of solutions, then given a vector $\vec{b}$ of the appropriate dimension, $A\vec{x} = \vec{b}$ has

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$\begin{bmatrix} 1 & \cancel{A} \\ 0 & 0 \end{bmatrix}$

- A. No solution
- B. Unique solution
- C. Infinitely many solutions
- D. at most one solution
- E. either no solution or an infinite number of solutions
- F. either a unique solution or an infinite number of solutions
- G. no solution, a unique solution or an infinite number of solutions, depending on the system of equations
- H. none of the above

Problem 3. A vector b is a linear combination of the columns of a matrix A if and only if the equation $Ax = b$ has at least one solution.

- A. True
- B. False

Problem 4.
Assume $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ does not span $\mathbb{R}^3$.
Select the best statement.

- A. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ always spans $\mathbb{R}^3$.
- B. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ spans $\mathbb{R}^3$ unless $\mathbf{u}_4$ is the zero vector.
- C. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ spans $\mathbb{R}^3$ unless $\mathbf{u}_4$ is a scalar multiple of another vector in the set.
- D. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ spans $\mathbb{R}^3$ unless $\mathbf{u}_4$ is in span$\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$.
- E. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ never spans $\mathbb{R}^3$.
- F. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ may, but does not have to, span $\mathbb{R}^3$.
- G. none of the above

Problem 5. Given that $v_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ is an eigenvector of the matrix $A = \begin{bmatrix} 8 & -3 \\ 18 & -7 \end{bmatrix}$, determine the corresponding eigenvalue.

- A. -4
- B. -3
- C. -2
- D. -1
- E. 0
- F. 1
- G. 2
- H. 3
- I. 4
- J. None of those above

$$\begin{bmatrix} 8 & -3 \\ 18 & -7 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 8-6 \\ 18-14 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$$

$$= 2\begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

Problem 6. Suppose A is a 4×2 matrix. Then *col* A is a subspace of R^k where $k =$

- A. -4
- B. -3
- C. -2
- D. -1
- E. 0
- F. 1
- G. 2
- H. 3
- I. 4
- J. none of the above

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4 $\begin{bmatrix} \times \\ \times \\ \times \\ \times \end{bmatrix}$

Problem 7. Let A be a matrix with linearly independent columns.
Select the best statement.

$\begin{bmatrix}1\\0\end{bmatrix}$

- A. The equation $A\mathbf{x} = \mathbf{b}$ has a solution for all $\mathbf{b}$ precisely when it has more rows than columns. X
- B. The equation $A\mathbf{x} = \mathbf{b}$ has a solution for all $\mathbf{b}$ precisely when it is a square matrix. $\Rightarrow$ pivot in all column $\Rightarrow$ pivot in all rows
- C. The equation $A\mathbf{x} = \mathbf{b}$ never has a solution for all $\mathbf{b}$. $\begin{bmatrix}1&0\\0&1\end{bmatrix}$
- D. The equation $A\mathbf{x} = \mathbf{b}$ has a solution for all $\mathbf{b}$ precisely when it has more columns than rows. X2 not l.i.
- E. There is insufficient information to determine if $A\mathbf{x} = \mathbf{b}$ has a solution for all $\mathbf{b}$.
- F. The equation $A\mathbf{x} = \mathbf{b}$ always has a solution for all $\mathbf{b}$. X
- G. none of the above

Problem 8. If $A = \begin{bmatrix} -1 & 4 \\ -7 & 5 \end{bmatrix}$, $B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$, and $AB = I$, the identity matrix, then $b_{11} =$

- A. $\frac{1}{23}$
- B. $\frac{4}{23}$
- C. $\frac{5}{23}$
- D. $\frac{7}{23}$
- E. $-\frac{1}{23}$
- F. $-\frac{4}{23}$
- G. $-\frac{5}{23}$
- H. $-\frac{7}{23}$
- I. 7
- J. None of those above

$$\begin{bmatrix} -1 & 4 \\ -7 & 5 \end{bmatrix} \begin{bmatrix} 5/23 & -4/23 \\ 7/23 & -1/23 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Problem 9.
$$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{bmatrix}$$

The determinant of the above matrix is

- A. -362880
- B. -40320
- C. -540
- D. -1
- E. 0
- F. 1
- G. 540
- H. 40320
- I. 362880
- J. None of the above

Problem 10. If $\vec{x}_1$ and $\vec{x}_2$ are solutions to $A\vec{x}=\vec{b}$, then $8\vec{x}_1 - 3\vec{x}_2$ is also a solution to $A\vec{x}=\vec{b}$.

- A. True
- B. False

Problem 11. The determinant of a square matrix A is nonzero if and only if the equation $Ax = 0$ has a unique solution.

- A. True
- B. False

Problem 12. Find the null space for $A = \begin{bmatrix} 1 & -4 \\ -1 & 4 \\ 4 & -16 \end{bmatrix}$. $\sim \begin{bmatrix} 1 & -4 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$

What is null(A)?

- A. span$\left\{\begin{bmatrix} 1 \\ -1 \\ 4 \end{bmatrix}\right\}$
- B. $\mathbb{R}^2$
- C. $\mathbb{R}^3$
- D. span$\left\{\begin{bmatrix} 4 \\ 1 \end{bmatrix}\right\}$
- E. span$\left\{\begin{bmatrix} -16 \\ 4 \end{bmatrix}\right\}$
- F. span$\left\{\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}\right\}$
- G. span$\left\{\begin{bmatrix} 0 \\ 0 \end{bmatrix}\right\}$
- H. none of the above

$x_1 = 4x_2$

$x_2 = x_2$

$\Rightarrow \bar{x} = \begin{bmatrix} 4 \\ 1 \end{bmatrix} x_2$

Check: $\begin{bmatrix} 1 & -4 \\ -1 & 4 \\ 4 & -16 \end{bmatrix} \begin{bmatrix} 4 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

Problem 13. Find all values of x for which $\text{rank}(A) = 2$. where A = $\begin{bmatrix} 1 & 2 & 0 & 7 \\ -2 & -2 & x & -5 \\ -3 & -8 & 2 & -30 \end{bmatrix}$

$+2 \quad +4 \quad +0 \quad +14$

$+3 \quad +6 \quad +0 \quad +21$

- A. -4
- B. -3
- C. -2
- D. -1
- E. 0
- F. 1
- G. 2
- H. 3
- I. 4
- J. none of the above

$$\begin{bmatrix} 1 & 2 & 0 & 7 \\ 0 & 2 & x & 9 \\ 0 & -2 & 2 & -9 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 0 & 7 \\ 0 & 2 & x & 9 \\ 0 & 0 & x+2 & 0 \end{bmatrix}$$

$x + 2 = 0$

$\Rightarrow x = -2$