Math 2550 Matrix Algebra Multiple Choice Exam **Form B**
Oct. 27, 2014

Problem 1. Suppose A is a square matrix and $A\vec{x} = \vec{0}$ has a unique solution, then given a vector $\vec{b}$ of the appropriate dimension, $A\vec{x} = \vec{b}$ has

- A. No solution
- B. Unique solution
- C. Infinitely many solutions
- D. at most one solution
- E. either no solution or an infinite number of solutions
- F. either a unique solution or an infinite number of solutions
- G. no solution, a unique solution or an infinite number of solutions, depending on the system of equations
- H. none of the above

eg $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Problem 2. Suppose that A is a 8×9 matrix which has a null space of dimension 6. The rank of A=

- A. -4
- B. -3
- C. -2
- D. -1
- E. 0
- F. 1
- G. 2
- H. 3
- I. 4
- J. none of the above

9

8 6

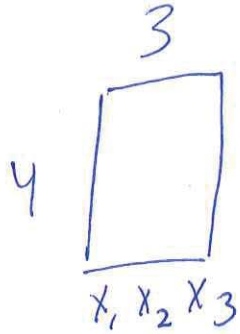
$9 - 6 = 3$

Problem 3. If $\vec{x}_1$ and $\vec{x}_2$ are solutions to $A\vec{x} = \vec{0}$, then $9\vec{x}_1 + 3\vec{x}_2$ is also a solution to $A\vec{x} = \vec{0}$.

- A. True
- B. False

Problem 4. Suppose A is a $4 \times \cancel{8}$ matrix. Then *nul* A is a subspace of R^k where $k =$

3



- A. -4
- B. -3
- C. -2
- D. -1
- E. 0
- F. 1
- G. 2
- H. 3
- I. 4
- J. none of the above

Problem 5. Let $\mathbf{u}_4$ be a linear combination of $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$. Select the best statement.

- A. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ is never a linearly dependent set of vectors.
- B. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ is a linearly dependent set of vectors unless one of $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ is the zero vector.
- C. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ is never a linearly independent set of vectors.
- D. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ is always a linearly independent set of vectors.
- E. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ is a linearly dependent set of vectors.
- F. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ could be a linearly dependent or linearly independent set of vectors depending on the vectors chosen.
- G. none of the above

Problem 6. The determinant of a square matrix A is 0 if and only if the equation $Ax = 0$ has an infinite number of solutions.

- A. True
- B. False

Problem 7. Let A be a matrix with linearly independent columns. Select the best statement.

- A. The equation $A\mathbf{x} = \mathbf{0}$ has nontrivial solutions precisely when it has more rows than columns.
- B. The equation $A\mathbf{x} = \mathbf{0}$ has nontrivial solutions precisely when it has more columns than rows.
- C. The ~~equation $A\mathbf{x} = \mathbf{0}$ has~~ nontrivial solutions precisely when it is a square matrix.
- D. The equation $A\mathbf{x} = \mathbf{0}$ never has nontrivial solutions.
- E. There is insufficient information to determine if such an equation has nontrivial solutions.
- F. The equation $A\mathbf{x} = \mathbf{0}$ always has nontrivial solutions.
- G. none of the above

Problem 8. Suppose $A \begin{bmatrix} 5 \\ 1 \\ -3 \end{bmatrix} = \begin{bmatrix} -15 \\ -3 \\ 9 \end{bmatrix}$. Then an eigenvalue of A is

x -3

- A. -4
- B. -3
- C. -2
- D. -1
- E. 0
- F. 1
- G. 2
- H. 3
- I. 4
- J. none of the above

Problem 9. Find the null space for $A = \begin{bmatrix} 1 & 0 & 6 \\ 0 & 1 & 3 \end{bmatrix}$.

What is $\mathrm{null}(A)$?

- A. $\mathrm{span}\left\{ \begin{bmatrix} 1 \\ 0 \\ -6 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -3 \end{bmatrix} \right\}$
- B. $\mathrm{span}\left\{ \begin{bmatrix} -3 \\ -6 \\ 1 \end{bmatrix} \right\}$
- C. $\mathbb{R}^3$
- D. $\mathbb{R}^2$
- E. $\mathrm{span}\left\{ \begin{bmatrix} -6 \\ -3 \\ 1 \end{bmatrix} \right\}$
- F. $\mathrm{span}\left\{ \begin{bmatrix} -6 \\ -3 \end{bmatrix} \right\}$
- G. $\mathrm{span}\left\{ \begin{bmatrix} -3 \\ -6 \end{bmatrix} \right\}$
- H. none of the above

3

Problem 10. $A = \begin{bmatrix} 2 & -1 \\ 0 & 1 \end{bmatrix}$. If $A^2 = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$, find b_{12}

- A. -4
- B. -3
- C. -2
- D. -1
- E. 0
- F. 1
- G. 2
- H. 3
- I. 4
- J. None of those above

$$\begin{bmatrix} 3 & -1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & -1 \\ 0 & 1 \end{bmatrix}$$

$$= -3 - 1$$

Problem 11. Find the volume of the parallelepiped determined by vectors $\begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix}$, $\begin{bmatrix} 0 \\ -2 \\ 0 \end{bmatrix}$, and $\begin{bmatrix} 0 \\ -5 \\ 1 \end{bmatrix}$

$$\begin{vmatrix} 2 & 0 & 0 \\ 0 & -2 & -5 \\ 2 & 0 & 1 \end{vmatrix} = 2 \begin{vmatrix} -2 & -5 \\ 0 & 1 \end{vmatrix} = 2(-2-0)$$

$$= -4$$

- A. -4
- B. -3
- C. -2
- D. -1
- E. 0
- F. 1
- G. 2
- H. 3
- I. 4
- J. None of those above

Problem 12. The vector $\vec{b}$ is in $ColA$ if and only if $A\vec{v} = \vec{b}$ has a solution

- A. True
- B. False

Problem 13. The vectors $v = \begin{bmatrix} -2 \\ -2 \\ 2 \end{bmatrix}$, $u = \begin{bmatrix} -3 \\ 7 \\ -17 \end{bmatrix}$, and $w = \begin{bmatrix} 3 \\ -3 \\ 8+k \end{bmatrix}$. are linearly independent if and only if $k \neq$ ____.

- A. -4
- B. -3
- C. -2
- D. -1
- E. 0
- F. 1
- G. 2
- H. 3
- I. 4
- J. none of the above

$$\begin{bmatrix} -2 & -3 & 3 \\ -2^{+2} & 7^{+3} & -3^{-3} \\ 2_{-2} & -17_{-3} & 8+K_{+3} \end{bmatrix}$$

$$\sim \begin{bmatrix} 2 & -3 & 3 \\ 0 & 10 & -6 \\ 0 & -20^{+20} & K+11^{-12} \end{bmatrix}$$

$$\sim \begin{bmatrix} 2 & -3 & 3 \\ 0 & 10 & -6 \\ 0 & 0 & K-1 \end{bmatrix}$$

$K-1=0$

$\Rightarrow K=1$