1. **(1 pt) Library/TCNJ/TCNJ_LinearSystems/problem3.pg**
Give a geometric description of the following systems of equations

? 1. $\begin{array}{rcrcr} -16x & + & 16y & = & -16 \\ -12x & + & 12y & = & -12 \\ -28x & + & 28y & = & -28 \end{array}$

? 2. $\begin{array}{rcrcr} 5x & + & y & = & 7 \\ 2x & - & 5y & = & 2 \\ 7x & + & 23y & = & 13 \end{array}$

? 3. $\begin{array}{rcrcr} 5x & + & y & = & 7 \\ 2x & - & 5y & = & 2 \\ 7x & + & 23y & = & 16 \end{array}$

Correct Answers:

- `Three identical lines`
- `Three lines intersecting at a single point`
- `Three non-parallel lines with no common intersection`

2. **(1 pt) Library/TCNJ/TCNJ_MatrixEquations/problem4.pg**

Let $A = \begin{bmatrix} 3 & -3 & 4 \\ -3 & -1 & -1 \\ -4 & -5 & 3 \end{bmatrix}$ and $x = \begin{bmatrix} -1 \\ 2 \\ 5 \end{bmatrix}$.

? 1. What does Ax mean?

Correct Answers:

- `Linear combination of the columns of A`

3. **(1 pt) Library/WHFreeman/Holt_linear_algebra/Chaps_1-4-/2.2.57.pg**

Assume $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ spans $\mathbb{R}^3$.
Select the best statement.

- A. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ spans $\mathbb{R}^3$ unless $\mathbf{u}_4$ is a scalar multiple of another vector in the set.
- B. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ never spans $\mathbb{R}^3$.
- C. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ spans $\mathbb{R}^3$ unless $\mathbf{u}_4$ is the zero vector.
- D. There is no easy way to determine if $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ spans $\mathbb{R}^3$.
- E. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ always spans $\mathbb{R}^3$.
- F. none of the above

Solution: *(Instructor solution preview: show the student solution after due date.)*

SOLUTION
The span of $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ is a subset of the span of $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$, so $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ always spans $\mathbb{R}^3$.

Correct Answers:

- `E`

4. **(1 pt) UI/Fall14/lin_span.pg**

Let $A = \begin{bmatrix} 3 \\ -8 \\ -7 \end{bmatrix}$, $B = \begin{bmatrix} 3 \\ -11 \\ -9 \end{bmatrix}$, and $C = \begin{bmatrix} -3 \\ 5 \\ 5 \end{bmatrix}$.

Which of the following best describes the span of the above 3 vectors?

- A. 0-dimensional point in R^3
- B. 1-dimensional line in R^3
- C. 2-dimensional plane in R^3
- D. R^3

Determine whether or not the three vectors listed above are linearly independent or linearly dependent.

- A. linearly dependent
- B. linearly independent

If they are linearly dependent, determine a non-trivial linear relation. Otherwise, if the vectors are linearly independent, enter 0's for the coefficients, since that relationship **always** holds.
____$A+$ ____$B+$ ____$C = 0$.

Correct Answers:

- `C`
- `A`
- `2; -1; 1`

5. **(1 pt) local/Library/TCNJ/TCNJ_BasesLinearlyIndependentSet-/3.pg**
Check the true statements below:

- A. The columns of an invertible $n \times n$ matrix form a basis for $\mathbb{R}^n$.
- B. If B is an echelon form of a matrix A, then the pivot columns of B form a basis for $ColA$.
- C. The column space of a matrix A is the set of solutions of $Ax = b$.
- D. If $H = Span\{b_1, ..., b_p\}$, then $\{b_1, ..., b_p\}$ is a basis for H.
- E. A basis is a spanning set that is as large as possible.

Correct Answers:

- `A`

6. **(1 pt) local/Library/UI/4.1.23.pg**

Find the null space for $A = \begin{bmatrix} 1 & 0 & -7 \\ 0 & 1 & -4 \end{bmatrix}$.

What is null(A)?

- A. span$\left\{ \begin{bmatrix} 1 \\ 0 \\ +7 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ +4 \end{bmatrix} \right\}$
- B. span$\left\{ \begin{bmatrix} +4 \\ +7 \end{bmatrix} \right\}$
- C. $\mathbb{R}^2$
- D. span$\left\{ \begin{bmatrix} +7 \\ +4 \\ 1 \end{bmatrix} \right\}$
- E. span$\left\{ \begin{bmatrix} +4 \\ +7 \\ 1 \end{bmatrix} \right\}$
- F. span$\left\{ \begin{bmatrix} +7 \\ +4 \end{bmatrix} \right\}$
- G. $\mathbb{R}^3$
- H. none of the above

.

Solution: *(Instructor solution preview: show the student solution after due date.)*

SOLUTION

A is row reduced. The basis of the null space has one element for each column without a leading one in the row reduced matrix.

Thus $A\mathbf{x} = \mathbf{0}$ has a one dimentional null space,

and thus, null(A) is the subspace generated by $\begin{bmatrix} 1-7 \\ 1-4 \\ 1 \end{bmatrix}$.

Correct Answers:

- D

7. **(1 pt) local/Library/UI/Fall14/HW7_12.pg**

Suppose that A is a 8×9 matrix which has a null space of dimension 5. The rank of A=

- A. -4
- B. -3
- C. -2
- D. -1
- E. 0
- F. 1
- G. 2
- H. 3
- I. 4
- J. none of the above

Solution: *(Instructor solution preview: show the student solution after due date.)*

SOLUTION

Using the Rank-Nullity theorem, if the dimensions of A is n x m, rank(A) = m - nullity(A) = 9 - 5 = 4

Correct Answers:

- I

8. **(1 pt) local/Library/UI/Fall14/HW8_5.pg**

Find the determinant of the matrix

$$A = \begin{bmatrix} -5 & 0 & 0 & 0 \\ 7 & 1 & 0 & 0 \\ -8 & 7 & 2 & 0 \\ -6 & 8 & 1 & -6 \end{bmatrix}.$$

$\det(A) =$

- A. -400
- B. -360
- C. -288
- D. -120
- E. 0
- F. 120
- G. 60
- H. 240
- I. 360
- J. 400
- K. None of those above

Correct Answers:

- G

If A is an $m \times n$ matrix and if the equation $Ax = b$ is inconsistent for some b in $\mathbb{R}^m$, then A cannot have a pivot position in every row.

- A. True
- B. False

Correct Answers:

- A

If the equation $Ax = b$ is inconsistent, then b is not in the set spanned by the columns of A.

- A. True
- B. False

Correct Answers:

- A

11. **(1 pt) local/Library/UI/Fall14/volume1.pg**

Find the volume of the parallelepiped determined by vectors $\begin{bmatrix} -5 \\ 0 \\ -3 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}$, and $\begin{bmatrix} -3 \\ 5 \\ 2 \end{bmatrix}$

- A. 38
- B. -5
- C. -4
- D. -2
- E. -1
- F. 0
- G. 1
- H. 3
- I. 5
- J. 7
- K. None of those above

Correct Answers:

- A

Suppose $A\vec{x} = \vec{0}$ has an infinite number of solutions, then given a vector $\vec{b}$ of the appropriate dimension, $A\vec{x} = \vec{b}$ has

- A. No solution
- B. Unique solution
- C. Infinitely many solutions
- D. at most one solution
- E. either no solution or an infinite number of solutions
- F. either a unique solution or an infinite number of solutions
- G. no solution, a unique solution or an infinite number of solutions, depending on the system of equations
- H. none of the above

Correct Answers:

- E

Suppose A is a square matrix and $A\vec{x} = \vec{0}$ has an infinite number of solutions, then given a vector $\vec{b}$ of the appropriate dimension, $A\vec{x} = \vec{b}$ has

- A. No solution
- B. Unique solution
- C. Infinitely many solutions
- D. at most one solution
- E. either no solution or an infinite number of solutions
- F. either a unique solution or an infinite number of solutions
- G. no solution, a unique solution or an infinite number of solutions, depending on the system of equations
- H. none of the above

Correct Answers:

- E

---

14. **(1 pt) local/Library/UI/problem7.pg**

A and B are $n \times n$ matrices.

Adding a multiple of one row to another does not affect the determinant of a matrix.

- A. True
- B. False

If the columns of A are linearly dependent, then $detA = 0$.

- A. True
- B. False

$det(A+B) = detA + detB$.

- A. True
- B. False

Correct Answers:

- A
- A
- B

The vector $\vec{b}$ is in $ColA$ if and only if $A\vec{v} = \vec{b}$ has a solution

- A. True
- B. False

Correct Answers:

- A

The vector $\vec{v}$ is in $NulA$ if and only if $A\vec{v} = \vec{0}$

- A. True
- B. False

Correct Answers:

- A

If $\vec{x}_1$ and $\vec{x}_2$ are solutions to $A\vec{x} = \vec{0}$, then $5\vec{x}_1 + 4\vec{x}_2$ is also a solution to $A\vec{x} = \vec{0}$.

- A. True
- B. False

Hint: *(Instructor hint preview: show the student hint after 0 attempts. The current number of attempts is 0.)*
Is $NulA$ a subspace? Is $NulA$ closed under linear combinations?

Correct Answers:

- A

If $\vec{x}_1$ and $\vec{x}_2$ are solutions to $A\vec{x} = \vec{b}$, then $-3\vec{x}_1 + 9\vec{x}_2$ is also a solution to $A\vec{x} = \vec{b}$.

- A. True
- B. False

Hint: *(Instructor hint preview: show the student hint after 0 attempts. The current number of attempts is 0.)*
Is the solution set to $A\vec{x} = \vec{b}$ a subspace even when $\vec{b}$ is not $\vec{0}$? Is the solution set to $A\vec{x} = \vec{b}$ closed under linear combinations even when $\vec{b}$ is not $\vec{0}$?

Correct Answers:

- B

Find the area of the parallelogram determined by the vectors $\begin{bmatrix} -6 \\ 4 \end{bmatrix}$ and $\begin{bmatrix} 4 \\ -2 \end{bmatrix}$.

- A. -4
- B. -3
- C. -2
- D. -1
- E. 0
- F. 1
- G. 2
- H. 3
- I. 4
- J. 5

Correct Answers:

- I

Suppose A is a 5×3 matrix. Then *nul* A is a subspace of R^k where $k =$

- A. -4
- B. -3
- C. -2
- D. -1
- E. 0
- F. 1
- G. 2
- H. 3
- I. 4
- J. none of the above

Correct Answers:

- H

Suppose A is a 4×7 matrix. Then *col* A is a subspace of R^k where $k =$

- A. -4
- B. -3
- C. -2
- D. -1
- E. 0
- F. 1
- G. 2
- H. 3
- I. 4
- J. none of the above

Correct Answers:

- I

---

22. (1 pt) Library/Rochester/setLinearAlgebra4InverseMatrix-/ur_la_4_2.pg

The matrix $\begin{bmatrix} 8 & 1 \\ 9 & k \end{bmatrix}$ is invertible if and only if $k \neq$____.

Correct Answers:

- 1.125

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23. (1 pt) Library/Rochester/setLinearAlgebra9Dependence-/ur_la_9_7.pg

The vectors
$v = \begin{bmatrix} -4 \\ 11 \\ -10 \end{bmatrix}$, $u = \begin{bmatrix} 2 \\ -4 \\ 9+k \end{bmatrix}$, and $w = \begin{bmatrix} 2 \\ -5 \\ 4 \end{bmatrix}$.
are linearly independent if and only if $k \neq$ _____.

Correct Answers:

- -7

---

24. (1 pt) Library/Rochester/setLinearAlgebra23QuadraticForms-/ur_la_23_2.pg

Find the eigenvalues of the matrix

$$M = \begin{bmatrix} 5 & 55 \\ 55 & 5 \end{bmatrix}.$$

Enter the two eigenvalues, separated by a comma:

____

Classify the quadratic form $Q(x) = x^T Ax$:

- A. $Q(x)$ is indefinite
- B. $Q(x)$ is positive definite

- C. $Q(x)$ is negative semidefinite
- D. $Q(x)$ is negative definite
- E. $Q(x)$ is positive semidefinite

Correct Answers:

- `-50, 60`
- `A`

---

25. **(1 pt) Library/Rochester/setLinearAlgebra23QuadraticForms-/ur_la_23_3.pg**

The matrix

$$A = \begin{bmatrix} -2.5 & -1.5 & 0 \\ -1.5 & -2.5 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

has three distinct eigenvalues, $\lambda_1 < \lambda_2 < \lambda_3$,

$\lambda_1 =$ ___,

$\lambda_2 =$ ___,

$\lambda_3 =$ ___.

Classify the quadratic form $Q(x) = x^T Ax$:

- A. $Q(x)$ is negative definite
- B. $Q(x)$ is positive semidefinite
- C. $Q(x)$ is negative semidefinite
- D. $Q(x)$ is positive definite
- E. $Q(x)$ is indefinite

Correct Answers:

- `-4`
- `-1`
- `2`
- `E`

---

26. **(1 pt) Library/TCNJ/TCNJ_BasesLinearlyIndependentSet-/problem5.pg**

Let W_1 be the set: $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$.

Determine if W_1 is a basis for $\mathbb{R}^3$ and check the correct answer(s) below.

- A. W_1 is not a basis because it does not span $\mathbb{R}^3$.
- B. W_1 is not a basis because it is linearly dependent.
- C. W_1 is a basis.

Let W_2 be the set: $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$.

Determine if W_2 is a basis for $\mathbb{R}^3$ and check the correct answer(s) below.

- A. W_2 is not a basis because it does not span $\mathbb{R}^3$.
- B. W_2 is not a basis because it is linearly dependent.
- C. W_2 is a basis.

Correct Answers:

- `C`
- `AB`

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27. **(1 pt) Library/WHFreeman/Holt_linear_algebra/Chaps_1-4-/2.3.42.pg**

Let A be a matrix with more columns than rows.
Select the best statement.

- A. The columns of A could be either linearly dependent or linearly independent depending on the case.
- B. The columns of A are linearly independent, as long as they does not include the zero vector.
- C. The columns of A are linearly independent, as long as no column is a scalar multiple of another column in A
- D. The columns of A must be linearly dependent.
- E. none of the above

Solution: *(Instructor solution preview: show the student solution after due date.)*

SOLUTION

Since there are more columns than rows, when we row reduce the matrix not all columns can have a leading 1.

The columns of A must be linearly dependent.

Correct Answers:

- `D`

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28. **(1 pt) Library/WHFreeman/Holt_linear_algebra/Chaps_1-4-/2.3.46.pg**

Let $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ be a linearly dependent set of vectors.
Select the best statement.

- A. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ is a linearly independent set of vectors unless $\mathbf{u}_4$ is a linear combination of other vectors in the set.
- B. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ could be a linearly independent or linearly dependent set of vectors depending on the vectors chosen.
- C. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ is always a linearly dependent set of vectors.
- D. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ is a linearly independent set of vectors unless $\mathbf{u}_4 = \mathbf{0}$.
- E. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ is always a linearly independent set of vectors.
- F. none of the above

Solution: *(Instructor solution preview: show the student solution after due date.)*

SOLUTION

If the zero vector is a nontrivial linear combination of a vectors in a smaller set, then it is also a nontrivial combination of vectors in a bigger set containing those vectors.

$\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ is always a linearly dependent set of vectors.

Correct Answers:

- C

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29. **(1 pt) Library/WHFreeman/Holt_linear_algebra/Chaps_1-4-/2.3.47.pg**

Let $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ be a linearly independent set of vectors. Select the best statement.

- A. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ is always a linearly independent set of vectors.
- B. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ could be a linearly independent or linearly dependent set of vectors depending on the vectors chosen.
- C. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ is never a linearly independent set of vectors.
- D. none of the above

Solution: *(Instructor solution preview: show the student solution after due date.)*

SOLUTION

If the zero vector cannot be written as a nontrivial linear combination of a vectors in a smaller set, then it is also not a nontrivial combination of vectors in a proper subset of those vectors.

$\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ is always a linearly independent set of vectors.

Correct Answers:

- A

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30. **(1 pt) Library/WHFreeman/Holt_linear_algebra/Chaps_1-4-/4.1.27.pg**

Find the null space for $A = \begin{bmatrix} 1 & 7 \\ 3 & 7 \\ -4 & -7 \end{bmatrix}$.

What is null(A)?

- A. $\mathbb{R}^3$
- B. $\mathbb{R}^2$
- C. span$\left\{ \begin{bmatrix} 7 \\ 1 \end{bmatrix} \right\}$
- D. span$\left\{ \begin{bmatrix} 1 \\ 3 \\ -4 \end{bmatrix} \right\}$
- E. span$\left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right\}$
- F. $\left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\}$
- G. span$\left\{ \begin{bmatrix} -7 \\ 1 \end{bmatrix} \right\}$
- H. span$\left\{ \begin{bmatrix} 1 \\ 7 \end{bmatrix} \right\}$
- I. none of the above

.

Solution: *(Instructor solution preview: show the student solution after due date.)*

SOLUTION

A is row reduces to $\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$. The basis of the null space has one element for each column without a leading one in the row reduced matrix.

Thus $A\mathbf{x} = \mathbf{0}$ has a zero dimentional null space,

and null(A) is the zero vector $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$.

Correct Answers:

- F

---

31. **(1 pt) Library/WHFreeman/Holt_linear_algebra/Chaps_1-4-/4.3.47.pg**

Indicate whether the following statement is true or false.

[?] 1. If A and B are equivalent matrices, then col(A) = col(B).

Solution: *(Instructor solution preview: show the student solution after due date.)*

SOLUTION:

FALSE. Consider $A = \begin{bmatrix} 2 & 3 & 4 \\ 2 & 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 3 & 4 \\ 0 & 0 & 0 \end{bmatrix}$

Correct Answers:

- F

---

32. **(1 pt) Library/maCalcDB/setLinearAlgebra3Matrices/ur_la_3_6.pg**

If A and B are 3×2 matrices, and C is a 6×3 matrix, which of the following are defined?

- A. AC
- B. CA
- C. C^T
- D. $A^T C^T$
- E. $A+B$
- F. $C+B$

Correct Answers:

- BCDE

---

33. **(1 pt) UI/DIAGtfproblem1.pg**

A, P and D are $n \times n$ matrices.

Check the true statements below:

- A. If A is invertible, then A is diagonalizable.
- B. A is diagonalizable if and only if A has n eigenvalues, counting multiplicities.
- C. If A is diagonalizable, then A has n distinct eigenvalues.
- D. A is diagonalizable if A has n distinct eigenvectors.

- E. A is diagonalizable if A has n distinct linearly independent eigenvectors.
- F. If A is symmetric, then A is orthogonally diagonalizable.
- G. If A is diagonalizable, then A is invertible.
- H. If A is diagonalizable, then A is symmetric.
- I. A is diagonalizable if $A = PDP^{-1}$ for some diagonal matrix D and some invertible matrix P.
- J. If A is symmetric, then A is diagonalizable.
- K. If A is orthogonally diagonalizable, then A is symmetric.
- L. If there exists a basis for $\mathbb{R}^n$ consisting entirely of eigenvectors of A, then A is diagonalizable.
- M. If $AP = PD$, with D diagonal, then the nonzero columns of P must be eigenvectors of A.

Correct Answers:

- `EFIJKLM`

---

34. **(1 pt) UI/Fall14/lin_span2.pg**

Which of the following sets of vectors span R^3?

- A. $\begin{bmatrix} -7 \\ 0 \\ 6 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ 8 \end{bmatrix}, \begin{bmatrix} -5 \\ 4 \\ 9 \end{bmatrix}$
- B. $\begin{bmatrix} 1 \\ 9 \end{bmatrix}, \begin{bmatrix} -6 \\ -1 \end{bmatrix}$
- C. $\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 8 \\ 2 \end{bmatrix}$
- D. $\begin{bmatrix} 8 \\ 3 \\ -9 \end{bmatrix}, \begin{bmatrix} 6 \\ -8 \\ -2 \end{bmatrix}, \begin{bmatrix} 14 \\ -5 \\ -11 \end{bmatrix}$
- E. $\begin{bmatrix} 6 \\ 14 \end{bmatrix}, \begin{bmatrix} 3 \\ 7 \end{bmatrix}$
- F. $\begin{bmatrix} -9 \\ 6 \end{bmatrix}, \begin{bmatrix} -8 \\ -2 \end{bmatrix}, \begin{bmatrix} -3 \\ -7 \end{bmatrix}$

Which of the following sets of vectors are linearly independent?

- A. $\begin{bmatrix} -7 \\ 0 \\ 6 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ 8 \end{bmatrix}, \begin{bmatrix} -5 \\ 4 \\ 9 \end{bmatrix}$
- B. $\begin{bmatrix} 1 \\ 9 \end{bmatrix}, \begin{bmatrix} -6 \\ -1 \end{bmatrix}$
- C. $\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 8 \\ 2 \end{bmatrix}$
- D. $\begin{bmatrix} 8 \\ 3 \\ -9 \end{bmatrix}, \begin{bmatrix} 6 \\ -8 \\ -2 \end{bmatrix}, \begin{bmatrix} 14 \\ -5 \\ -11 \end{bmatrix}$
- E. $\begin{bmatrix} 6 \\ 14 \end{bmatrix}, \begin{bmatrix} 3 \\ 7 \end{bmatrix}$
- F. $\begin{bmatrix} -9 \\ 6 \end{bmatrix}, \begin{bmatrix} -8 \\ -2 \end{bmatrix}, \begin{bmatrix} -3 \\ -7 \end{bmatrix}$

Correct Answers:

- `A`
- `AB`

---

35. **(1 pt) UI/orthog.pg**

All vectors and subspaces are in $\mathbb{R}^n$.

Check the true statements below:

- A. If A is symmetric, $A\mathbf{v} = r\mathbf{v}$, $A\mathbf{w} = s\mathbf{w}$ and $r \neq s$, then $\mathbf{v} \cdot \mathbf{w} = 0$.
- B. If $W = Span\{x_1, x_2, x_3\}$ and if $\{v_1, v_2, v_3\}$ is an orthonormal set in W, then $\{v_1, v_2, v_3\}$ is an orthonormal basis for W.
- C. If $A\mathbf{v} = r\mathbf{v}$ and $A\mathbf{w} = s\mathbf{w}$ and $r \neq s$, then $\mathbf{v} \cdot \mathbf{w} = 0$.
- D. If x is not in a subspace W, then $x - \text{proj}_W(x)$ is not zero.
- E. If $\{v_1, v_2, v_3\}$ is an orthonormal set, then the set $\{v_1, v_2, v_3\}$ is linearly independent.
- F. In a QR factorization, say $A = QR$ (when A has linearly independent columns), the columns of Q form an orthonormal basis for the column space of A.
- G. If $\mathbf{v}$ and $\mathbf{w}$ are both eigenvectors of A and if A is symmetric, then $\mathbf{v} \cdot \mathbf{w} = 0$.

Correct Answers:

- `ABDEF`

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36. **(1 pt) local/Library/UI/2.3.49.pg**

Let $\mathbf{u}_4$ be a linear combination of $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$.
Select the best statement.

- A. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ could be a linearly dependent or linearly independent set of vectors depending on the vectors chosen.
- B. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ is never a linearly independent set of vectors.
- C. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ is a linearly dependent set of vectors.
- D. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ is always a linearly independent set of vectors.
- E. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ is a linearly dependent set of vectors unless one of $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ is the zero vector.
- F. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ is never a linearly dependent set of vectors.
- G. none of the above

Solution: *(Instructor solution preview: show the student solution after due date.)*

SOLUTION

If $\mathbf{u}_4 = a_1\mathbf{u}_1 + a_2\mathbf{u}_2 + a_3\mathbf{u}_3$, then
$0 = a_1\mathbf{u}_1 + a_2\mathbf{u}_2 + a_3\mathbf{u}_3 - \mathbf{u}_4$

"$\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ is never a linearly independent set of vectors."

Correct Answers:

- B

---

37. **(1 pt) local/Library/UI/Fall14/HW7_6.pg**

If A is an $n \times n$ matrix and $\mathbf{b} \neq 0$ in $\mathbb{R}^n$, then consider the set of solutions to $A\mathbf{x} = \mathbf{b}$.

Select true or false for each statement.

The set contains the zero vector

- A. True
- B. False

This set is closed under vector addition

- A. True
- B. False

This set is closed under scalar multiplications

- A. True
- B. False

This set is a subspace

- A. True
- B. False

Solution: *(Instructor solution preview: show the student solution after due date.)*

SOLUTION

$A\mathbf{0} = \mathbf{0} \neq \mathbf{b}$, so the zero vector is not in the set and it is not a subspace.

Correct Answers:

- B
- B
- B
- B

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38. **(1 pt) local/Library/UI/Fall14/HW7_11.pg**

Find all values of x for which $\text{rank}(A) = 2$.

$$\text{A} = \begin{bmatrix} 1 & 1 & 0 & 7 \\ 2 & 4 & \text{x} & 21 \\ 1 & 7 & 6 & 28 \end{bmatrix}$$

$x =$

- A. -4
- B. -3
- C. -2
- D. -1
- E. 0
- F. 1
- G. 2
- H. 3
- I. 4
- J. none of the above

Solution: *(Instructor solution preview: show the student solution after due date.)*

SOLUTION:

Row reduce A to get:

$$\begin{bmatrix} 1 & 1 & 0 & 7 \\ 2 & 4 & x & 21 \\ 1 & 7 & 6 & 28 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 0 & 7 \\ 0 & 2 & x & 7 \\ 0 & 6 & 6 & 21 \end{bmatrix}$$

Since two pivots are needed, $x = 2$

Correct Answers:

- G

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39. **(1 pt) local/Library/UI/Fall14/HW8_7.pg**

Suppose that a 4×4 matrix A with rows v_1, v_2, v_3, and v_4 has determinant $\det A = 6$. Find the following determinants:

$$B = \begin{bmatrix} v_1 \\ v_2 \\ 9v_3 \\ v_4 \end{bmatrix} \det(B) =$$

- A. -18
- B. -15
- C. -12
- D. 54
- E. -9
- F. 0
- G. 9
- H. 12
- I. 15
- J. 18
- K. None of those above

$$C = \begin{bmatrix} v_4 \\ v_3 \\ v_2 \\ v_1 \end{bmatrix} \det(C) =$$

- A. -18
- B. 6
- C. -9
- D. -3
- E. 0
- F. 3
- G. 9
- H. 12
- I. 18
- J. None of those above

$$D = \begin{bmatrix} v_1 + 3v_3 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix}$$

$\det(D) =$

- A. -18
- B. 6
- C. -9
- D. -3
- E. 0
- F. 3

- G. 9
- H. 12
- I. 18
- J. None of those above

Correct Answers:

- D
- B
- B

A vector b is a linear combination of the columns of a matrix A if and only if the equation $Ax = b$ has at least one solution.

- A. True
- B. False

Correct Answers:

- A

Any linear combination of vectors can always be written in the form Ax for a suitable matrix A and vector x.

- A. True
- B. False

Correct Answers:

- A

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42. **(1 pt) local/Library/UI/Fall14/quiz2_9.pg**

Supppose A is an invertible $n \times n$ matrix and v is an eigenvector of A with associated eigenvalue -5. Convince yourself that v is an eigenvector of the following matrices, and find the associated eigenvalues:

1. A^4, eigenvalue =

- A. 16
- B. 81
- C. 125
- D. 216
- E. 1024
- F. 625
- G. 2000
- H. None of those above

2. A^{-1}, eigenvalue =

- A. -0.5
- B. -0.333
- C. -0.2
- D. -0.125
- E. 0
- F. 0.125
- G. 0.333
- H. 0.5
- I. None of those above

3. $A + 9I_n$, eigenvalue =

- A. -8
- B. -4
- C. -5
- D. 0
- E. 2
- F. 4
- G. 10
- H. None of those above

4. $6A$, eigenvalue =

- A. -40
- B. -36
- C. -28
- D. -30
- E. -12
- F. 0
- G. 24
- H. 36
- I. None of those above

Correct Answers:

- F
- C
- F
- D

---

43. **(1 pt) local/Library/UI/Fall14/quiz2_10.pg**

If $v_1 = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$ and $v_2 = \begin{bmatrix} -2 \\ -1 \end{bmatrix}$

are eigenvectors of a matrix A corresponding to the eigenvalues $\lambda_1 = 1$ and $\lambda_2 = -4$, respectively, then

a. $A(v_1 + v_2) =$

- A. $\begin{bmatrix} -3 \\ 5 \end{bmatrix}$
- B. $\begin{bmatrix} -2 \\ 4 \end{bmatrix}$
- C. $\begin{bmatrix} -6 \\ 5 \end{bmatrix}$
- D. $\begin{bmatrix} 10 \\ 6 \end{bmatrix}$
- E. $\begin{bmatrix} 12 \\ 4 \end{bmatrix}$
- F. $\begin{bmatrix} 11 \\ 3 \end{bmatrix}$
- G. None of those above

b. $A(3v_1) =$

- A. $\begin{bmatrix} -12 \\ -12 \end{bmatrix}$
- B. $\begin{bmatrix} -2 \\ 8 \end{bmatrix}$
- C. $\begin{bmatrix} -6 \\ 4 \end{bmatrix}$
- D. $\begin{bmatrix} 10 \\ 6 \end{bmatrix}$
- E. $\begin{bmatrix} 9 \\ -3 \end{bmatrix}$
- F. $\begin{bmatrix} 12 \\ 4 \end{bmatrix}$
- G. $\begin{bmatrix} 11 \\ 3 \end{bmatrix}$
- H. None of those above

Correct Answers:

- `F`
- `E`

---

44. **(1 pt) local/Library/UI/Fall14/quiz2_11.pg**

Let $v_1 = \begin{bmatrix} 0 \\ -2 \\ -1 \end{bmatrix}$, $v_2 = \begin{bmatrix} -3 \\ 1 \\ 0 \end{bmatrix}$, and $v_3 = \begin{bmatrix} 3 \\ 0 \\ -3 \end{bmatrix}$ be eigenvectors of the matrix A which correspond to the eigenvalues $\lambda_1 = -3$, $\lambda_2 = 2$, and $\lambda_3 = 4$, respectively, and let $v = \begin{bmatrix} 3 \\ 5 \\ -4 \end{bmatrix}$.

Express v as a linear combination of v_1, v_2, and v_3, and find Av.

1. If $v = c_1v_1 + c_2v_2 + c_3v_3$, then (c_1, c_2, c_3)=

- A. (1,2,2)
- B. (-3,2,4)
- C. (-4,7,3)
- D. (-2,1,2)
- E. (0,1,2)
- F. (4,-1,5)
- G. None of above

2. $Av =$

- A. $\begin{bmatrix} -12 \\ 7 \\ -12 \end{bmatrix}$
- B. $\begin{bmatrix} -2 \\ 12 \\ 8 \end{bmatrix}$
- C. $\begin{bmatrix} -6 \\ 7 \\ 4 \end{bmatrix}$
- D. $\begin{bmatrix} 10 \\ 0 \\ 6 \end{bmatrix}$
- E. $\begin{bmatrix} 18 \\ -10 \\ -30 \end{bmatrix}$
- F. $\begin{bmatrix} 12 \\ 8 \\ 4 \end{bmatrix}$
- G. $\begin{bmatrix} -7 \\ -3 \\ 12 \end{bmatrix}$
- H. None of those above

Correct Answers:

- `D`
- `E`

Suppose a coefficient matrix A contains a pivot in every row. Then $A\vec{x} = \vec{b}$ has

- A. No solution
- B. Unique solution
- C. Infinitely many solutions
- D. at most one solution
- E. either no solution or an infinite number of solutions
- F. either a unique solution or an infinite number of solutions
- G. no solution, a unique solution or an infinite number of solutions, depending on the system of equations
- H. none of the above

Correct Answers:

- `F`

Suppose a coefficient matrix A contains a pivot in every column. Then $A\vec{x} = \vec{b}$ has

- A. No solution
- B. Unique solution
- C. Infinitely many solutions
- D. at most one solution
- E. either no solution or an infinite number of solutions
- F. either a unique solution or an infinite number of solutions
- G. no solution, a unique solution or an infinite number of solutions, depending on the system of equations
- H. none of the above

Correct Answers:

- `D`

47. **(1 pt) local/Library/UI/MatrixAlgebra/Euclidean/2.3.43.pg**

Let A be a matrix with linearly independent columns.
Select the best statement.

- A. The equation $A\mathbf{x} = \mathbf{0}$ never has nontrivial solutions.
- B. The equation $A\mathbf{x} = \mathbf{0}$ has nontrivial solutions precisely when it is a square matrix.
- C. There is insufficient information to determine if such an equation has nontrivial solutions.
- D. The equation $A\mathbf{x} = \mathbf{0}$ has nontrivial solutions precisely when it has more columns than rows.
- E. The equation $A\mathbf{x} = \mathbf{0}$ has nontrivial solutions precisely when it has more rows than columns.
- F. The equation $A\mathbf{x} = \mathbf{0}$ always has nontrivial solutions.
- G. none of the above

Solution: *(Instructor solution preview: show the student solution after due date.)*

SOLUTION

The linear independence of the columns does not change with row reduction. Since the columns are linearly independent, after row reduction, each column contains a leading 1. We get nontrivial solutions when we have columns without a leading 1 in the row reduced matrix.

The equation $A\mathbf{x} = \mathbf{0}$ never has nontrivial solutions.

Correct Answers:

- `A`

---

48. **(1 pt) local/Library/UI/eigenTF.pg**

A is $n \times n$ an matrices.

Check the true statements below:

- A. 0 is an eigenvalue of A if and only if $Ax = 0$ has a nonzero solution
- B. 0 can never be an eigenvalue of A.
- C. The vector $\mathbf{0}$ is an eigenvector of A if and only if det(A) = 0
- D. The vector $\mathbf{0}$ can never be an eigenvector of A
- E. There are an infinite number of eigenvectors that correspond to a particular eigenvalue of A.
- F. 0 is an eigenvalue of A if and only if the columns of A are linearly dependent.
- G. 0 is an eigenvalue of A if and only if det(A) = 0
- H. A will have at most n eigenvalues.
- I. The eigenspace corresponding to a particular eigenvalue of A contains an infinite number of vectors.
- J. A will have at most n eigenvectors.
- K. The vector $\mathbf{0}$ is an eigenvector of A if and only if the columns of A are linearly dependent.
- L. The vector $\mathbf{0}$ is an eigenvector of A if and only if $Ax = 0$ has a nonzero solution
- M. 0 is an eigenvalue of A if and only if $Ax = 0$ has an infinite number of solutions

Correct Answers:

- `ADEFGHIM`

If $\vec{v}_1$ and $\vec{v}_2$ are eigenvectors of A corresponding to eigenvalue λ_0, then $6\vec{v}_1 + 8\vec{v}_2$ is also an eigenvector of A corresponding to eigenvalue λ_0 when $6\vec{v}_1 + 8\vec{v}_2$ is not $\vec{0}$.

- A. True
- B. False

Hint: *(Instructor hint preview: show the student hint after 0 attempts. The current number of attempts is 0.)*
Is an eigenspace a subspace? Is an eigenspace closed under linear combinations?
Also, is $6\vec{v}_1 + 8\vec{v}_2$ nonzero?

Correct Answers:

- `A`

Use Cramer's rule to solve the following system of equations for x:

$$\begin{aligned} 4x - 2y &= -14 \\ -1x + 1y &= 4 \end{aligned}$$

.

- A. -4
- B. -3
- C. -2
- D. -1
- E. 0
- F. 1
- G. 2
- H. 3
- I. 4
- J. none of the above

Correct Answers:

- `B`

Let $A = \begin{bmatrix} 3 & -9 & -4 \\ 0 & 7 & 5 \\ 0 & 0 & 3 \end{bmatrix}$. Is A = diagonalizable?

- A. yes
- B. no
- C. none of the above

Solution: *(Instructor solution preview: show the student solution after due date.)*

SOLUTION

Note that since the matrix is triangular, the eigenvalues are the diagonal elements 3 and 7. Since A is a 3 x 3 matrix, we need 3 linearly independent eigenvectors. Since 7 has algebraic multiplicity 1, it has geometric multiplicity 1 (the dimension of its eigenspace is 1). Thus we can only use one eigenvector from the eigenspace corresponding to eigenvalue 7 to form P.

Thus we need 2 linearly independent eigenvectors from the eigenspace corresponding to the other eigenvalue 3. The eigenvalue 3 has algebraic multiplicity 2. Let E = dimension of the eigenspace corresponding eigenvalue 3. Then $1 \leq E \leq 2$. But we can easily see that the Nullspace of $A - 3I$ has dimension 1.

Thus we do not have enough linearly independent eigenvectors to form P. Hence A is not diagonalizable.

Correct Answers:

- B

Let $A = \begin{bmatrix} -5 & -18 & -9 \\ 0 & 1 & 3 \\ 0 & 0 & -5 \end{bmatrix}$. Is A = diagonalizable?

- A. yes
- B. no
- C. none of the above

Solution: *(Instructor solution preview: show the student solution after due date.)*

SOLUTION

Note that since the matrix is triangular, the eigenvalues are the diagonal elements -5 and 1. Since A is a 3 x 3 matrix, we need 3 linearly independent eigenvectors. Since 1 has algebraic multiplicity 1, it has geometric multiplicity 1 (the dimension of its eigenspace is 1). Thus we can only use one eigenvector from the eigenspace corresponding to eigenvalue 1 to form P.

Thus we need 2 linearly independent eigenvectors from the eigenspace corresponding to the other eigenvalue -5. The eigenvalue -5 has algebraic multiplicity 2. Let E = dimension of the eigenspace corresponding eigenvalue -5. Then $1 \leq E \leq 2$. But we can easily see that the Nullspace of $A + 5I$ has dimension 2.

Thus we have 3 linearly independent eigenvectors which we can use to form the square matrix P. Hence A is diagonalizable.

Correct Answers:

- A

Let $A = \begin{bmatrix} 3.55384615384615 & -0.138461538461538 & 2.15604395604396 \\ 3.13846153846154 & 2.21538461538462 & -4.90549450549451 \\ 6.46153846153846 & -1.61538461538462 & -3.50769230769231 \end{bmatrix}$

and let $P = \begin{bmatrix} -1 & 9 & 5 \\ -4 & -2 & -7 \\ 0 & 7 & -7 \end{bmatrix}$.

Suppose $A = PDP^{-1}$. Then if d_{ii} are the diagonal entries of D, $d_{11} =$,

- A. -4
- B. -3
- C. -2
- D. -1
- E. 0
- F. 1
- G. 2
- H. 3
- I. 4
- J. 5

Hint: *(Instructor hint preview: show the student hint after 0 attempts. The current number of attempts is 0.)*

Use definition of eigenvalue since you know an eigenvector corresponding to eigenvalue d_{11}.

Correct Answers:

- H

Calculate the determinant of $\begin{bmatrix} 2.55555555555556 & 5 \\ 5 & 9 \end{bmatrix}$.

- A. -4
- B. -3
- C. -2
- D. -1
- E. 0
- F. 1
- G. 2
- H. 3
- I. 4
- J. 5

Correct Answers:

- C

Suppose $A \begin{bmatrix} 5 \\ 4 \\ -1 \end{bmatrix} = \begin{bmatrix} -10 \\ -8 \\ 2 \end{bmatrix}$. Then an eigenvalue of A is

- A. -4
- B. -3
- C. -2
- D. -1
- E. 0
- F. 1

- G. 2
- H. 3
- I. 4
- J. none of the above

Correct Answers:

- C

Suppose u and v are eigenvectors of A with eigenvalue 2 and w is an eigenvector of A with eigenvalue 3. Determine which of the following are eigenvectors of A and their corresponding eigenvalues.

(a.) If $4v$ an eigenvector of A, determine its eigenvalue. Else state it is not an eigenvector of A.

- A. -4
- B. -3
- C. -2
- D. -1
- E. 0
- F. 1
- G. 2
- H. 3
- I. 4
- J. $4v$ need not be an eigenvector of A

(b.) If $7u + 4v$ an eigenvector of A, determine its eigenvalue. Else state it is not an eigenvector of A.

- A. -4
- B. -3
- C. -2
- D. -1
- E. 0
- F. 1
- G. 2
- H. 3
- I. 4
- J. $7u + 4v$ need not be an eigenvector of A

(c.) If $7u + 4w$ an eigenvector of A, determine its eigenvalue. Else state it is not an eigenvector of A.

- A. -4
- B. -3
- C. -2
- D. -1
- E. 0
- F. 1
- G. 2
- H. 3
- I. 4
- J. $7u + 4w$ need not be an eigenvector of A

Correct Answers:

- G
- G
- J

If the characteristic polynomial of $A = (\lambda+6)^1(\lambda-7)^2(\lambda+6)^2$, then the algebraic multiplicity of $\lambda = 7$ is

- A. 0
- B. 1
- C. 2
- D. 3
- E. 0 or 1
- F. 0 or 2
- G. 1 or 2
- H. 0, 1, or 2
- I. 0, 1, 2, or 3
- J. none of the above

Correct Answers:

- C

If the characteristic polynomial of $A = (\lambda-4)^5(\lambda+3)^2(\lambda-1)^8$, then the geometric multiplicity of $\lambda = -3$ is

- A. 0
- B. 1
- C. 2
- D. 3
- E. 0 or 1
- F. 0 or 2
- G. 1 or 2
- H. 0, 1, or 2
- I. 0, 1, 2, or 3
- J. none of the above

Correct Answers:

- G