1. **(1 pt) Library/Rochester/setLinearAlgebra11Eigenvalues-/ur_la_11_18.pg**

The matrix $A = \begin{bmatrix} 3 & 0 & 2 \\ 0 & 1 & 0 \\ -2 & 0 & -1 \end{bmatrix}$

has one real eigenvalue. Find this eigenvalue and a basis of the eigenspace.

eigenvalue = ____,

Basis: $\begin{bmatrix} \text{—} \\ \text{—} \\ \text{—} \end{bmatrix}$, $\begin{bmatrix} \text{—} \\ \text{—} \\ \text{—} \end{bmatrix}$.

Correct Answers:

- 1
- \(\displaystyle\left.\begin{array}{c}
\mbox{-1} \cr
\mbox{1} \cr
\mbox{1} \cr
\end{array}\right.\) ,\(\displaystyle\left.\begin{array}{c}
\mbox{1} \cr
\mbox{0} \cr
\mbox{-1} \cr
\end{array}\right.\)

2. **(1 pt) Library/Rochester/setLinearAlgebra12Diagonalization-/ur_la_12_2.pg**

Let $M = \begin{bmatrix} 8 & -4 \\ 8 & -4 \end{bmatrix}$.

Find formulas for the entries of M^n, where n is a positive integer.

$M^n = \begin{bmatrix} \_\_\_\_ & \_\_\_\_ \\ \_\_\_\_ & \_\_\_\_ \end{bmatrix}$.

Correct Answers:

- 2*(4**n)
- \- 1*1*(4**n)
- 1*2*(4**n)
- \- 4**n

3. **(1 pt) Library/Rochester/setLinearAlgebra14TransfOfRn-/ur_la_14_18.pg**

Let L be the line in $\mathbb{R}^3$ that consists of all scalar multiples of the vector $\begin{bmatrix} -2 \\ 2 \\ 1 \end{bmatrix}$. Find the orthogonal projection of the vector $v = \begin{bmatrix} 9 \\ 3 \\ 8 \end{bmatrix}$ onto L.

$\text{proj}_L v = \begin{bmatrix} \_\_\_\_ \\ \_\_\_\_ \\ \_\_\_\_ \end{bmatrix}$.

Correct Answers:

- 0.888888888888889
- -0.888888888888889
- -0.444444444444444

4. **(1 pt) Library/Rochester/setLinearAlgebra17DotProductRn-/ur_la_17_6.pg**

Find a vector v perpendicular to the vector $u = \begin{bmatrix} -4 \\ 2 \end{bmatrix}$.

$v = \begin{bmatrix} \text{—} \\ \text{—} \end{bmatrix}$.

Correct Answers:

- \(\displaystyle\left.\begin{array}{c}
\mbox{2} \cr
\mbox{4} \cr
\end{array}\right.\)

5. **(1 pt) Library/Rochester/setLinearAlgebra17DotProductRn-/ur_la_17_7.pg**

Find the value of k for which the vectors

$x = \begin{bmatrix} -4 \\ -4 \\ -5 \\ 4 \end{bmatrix}$ and $y = \begin{bmatrix} 4 \\ 5 \\ -2 \\ k \end{bmatrix}$ are orthogonal.

$k =$ ____.

Correct Answers:

- 6.5

6. **(1 pt) Library/Rochester/setLinearAlgebra17DotProductRn-/ur_la_17_21.pg**

Let $v_1 = \begin{bmatrix} 0.5 \\ 0.5 \\ -0.5 \\ 0.5 \end{bmatrix}$, $v_2 = \begin{bmatrix} 0.5 \\ 0.5 \\ 0.5 \\ -0.5 \end{bmatrix}$, and $v_3 = \begin{bmatrix} -0.5 \\ 0.5 \\ 0.5 \\ 0.5 \end{bmatrix}$.

Find a vector v_4 in $\mathbb{R}^4$ such that the vectors v_1, v_2, v_3, and v_4 are orthonormal.

$v_4 = \begin{bmatrix} \text{—} \\ \text{—} \\ \text{—} \\ \text{—} \end{bmatrix}$.

Correct Answers:

- \(\displaystyle\left.\begin{array}{c}
\mbox{-0.5} \cr
\mbox{0.5} \cr
\mbox{-0.5} \cr
\mbox{-0.5} \cr
\end{array}\right.\)

7. **(1 pt) Library/Rochester/setLinearAlgebra18OrthogonalBases-/ur_la_18_4.pg**

Let $x = \begin{bmatrix} -2 \\ -3 \\ 1 \\ 0 \end{bmatrix}$ and $y = \begin{bmatrix} 0 \\ -3 \\ 5 \\ 4 \end{bmatrix}$.

Use the Gram-Schmidt process to determine an orthonormal basis for the subspace of $\mathbb{R}^4$ spanned by x and y.

$$\left[\begin{array}{c} \_\_\_ \\ \_\_\_ \\ \_\_\_ \\ \_\_\_ \end{array}\right], \left[\begin{array}{c} \_\_\_ \\ \_\_\_ \\ \_\_\_ \\ \_\_\_ \end{array}\right].$$

Correct Answers:

- -0.534522483824849
- -0.801783725737273
- 0.267261241912424
- 0
- 0.333333333333333
- 0
- 0.666666666666667
- 0.666666666666667

---

8. **(1 pt) Library/Rochester/setLinearAlgebra22SymmetricMatrices-/ur_la_22_3.pg**

Find the eigenvalues and associated unit eigenvectors of the (symmetric) matrix

$A = \left[\begin{array}{cc} 4 & -12 \\ -12 & 36 \end{array}\right]$.

smaller eigenvalue = ___,

associated unit eigenvector = $\left[\begin{array}{c} \_\_ \\ \_\_ \end{array}\right]$,

larger eigenvalue = ___,

associated unit eigenvector = $\left[\begin{array}{c} \_\_ \\ \_\_ \end{array}\right]$.

The above eigenvectors form an orthonormal eigenbasis for A.

Correct Answers:

- 0
- \(\displaystyle\left.\begin{array}{c}
\mbox{0.948683298050514} \cr
\mbox{0.316227766016838} \cr
\end{array}\right.\)
- 40
- \(\displaystyle\left.\begin{array}{c}
\mbox{-0.316227766016838} \cr
\mbox{0.948683298050514} \cr
\end{array}\right.\)

---

9. **(1 pt) Library/TCNJ/TCNJ_OrthogonalSets/problem9.pg**

Given $v = \left[\begin{array}{c} 3 \\ 1 \end{array}\right]$, find the coordinates for v in the subspace W spanned by $u_1 = \left[\begin{array}{c} 7 \\ -1 \end{array}\right]$ and $u_2 = \left[\begin{array}{c} 6 \\ 42 \end{array}\right]$. Note that u_1 and u_2 are orthogonal.

$v =$ ___ $u_1 +$ ___ u_2

Correct Answers:

- 0.4
- 0.0333333333333333

---

10. **(1 pt) Library/Rochester/setLinearAlgebra18OrthogonalBases-/ur_la_18_7.pg**

Let $A = \left[\begin{array}{cccc} -2 & 9 & 6 & 6 \\ 3 & 6 & -9 & 4 \end{array}\right]$.

Find an orthonormal basis of the kernel of A.

$$\left[\begin{array}{c} \_\_ \\ \_\_ \\ \_\_ \\ \_\_ \end{array}\right], \left[\begin{array}{c} \_\_ \\ \_\_ \\ \_\_ \\ \_\_ \end{array}\right].$$

Correct Answers:

- \(\displaystyle\left.\begin{array}{c}
\mbox{0.948683298050514} \cr
\mbox{0} \cr
\mbox{0.316227766016838} \cr
\mbox{0} \cr
\end{array}\right.\) ,\(\displaystyle\left.\begin{array}{c
\mbox{0} \cr
\mbox{-0.554700196225229} \cr
\mbox{0} \cr
\mbox{0.832050294337844} \cr
\end{array}\right.\)

---

11. **(1 pt) Library/Rochester/setLinearAlgebra18OrthogonalBases-/ur_la_18_11.pg**

Let $A = \left[\begin{array}{ccc} 1 & 0 & -4 \\ -2 & -4 & 4 \\ 2 & 5 & -3 \end{array}\right]$.

Find an orthonormal basis of the column space of A.

$$\left[\begin{array}{c} \_\_ \\ \_\_ \\ \_\_ \end{array}\right], \left[\begin{array}{c} \_\_ \\ \_\_ \\ \_\_ \end{array}\right].$$

Correct Answers:

- \(\displaystyle\left.\begin{array}{c}
\mbox{0.333333333333333} \cr
\mbox{-0.666666666666667} \cr
\mbox{0.666666666666667} \cr
\end{array}\right.\) ,\(\displaystyle\left.\begin{array}{c
\mbox{-0.894427190999916} \cr
\mbox{0} \cr
\mbox{0.447213595499958} \cr
\end{array}\right.\)

---

12. **(1 pt) Library/Rochester/setLinearAlgebra19QRfactorization-/ur_la_19_3.pg**

Find the QR factorization of $M = \left[\begin{array}{ccc} 4 & -4 & 4 \\ 2 & -5 & 11 \\ 4 & 2 & 4 \end{array}\right]$.

$$M = \left[\begin{array}{ccc} \_\_\_ & \_\_\_ & \_\_\_ \\ \_\_\_ & \_\_\_ & \_\_\_ \\ \_\_\_ & \_\_\_ & \_\_\_ \end{array}\right] \left[\begin{array}{ccc} \_\_\_ & \_\_\_ & \_\_\_ \\ \_\_\_ & \_\_\_ & \_\_\_ \\ \_\_\_ & \_\_\_ & \_\_\_ \end{array}\right].$$

Correct Answers:

- 0.666666666666667
- -0.333333333333333
- -0.666666666666667
- 0.333333333333333
- -0.666666666666667
- 0.666666666666667
- 0.666666666666667
- 0.666666666666667
- 0.333333333333333
- 6
- -3

- 9
- 0
- 6
- -6
- 0
- 0
- 6

13. **(1 pt) Library/Rochester/setLinearAlgebra19QRfactorization-/ur_la_19_4.pg**

Find the QR factorization of $M = \begin{bmatrix} 2 & 8 \\ -2 & 4 \\ -2 & -8 \\ -2 & 4 \end{bmatrix}$.

$M = \begin{bmatrix} \_\_\_ & \_\_\_ \\ \_\_\_ & \_\_\_ \\ \_\_\_ & \_\_\_ \\ \_\_\_ & \_\_\_ \end{bmatrix} \begin{bmatrix} \_\_\_ & \_\_\_ \\ \_\_\_ & \_\_\_ \end{bmatrix}$.

Correct Answers:

- 0.5
- 0.5
- -0.5
- 0.5
- -0.5
- -0.5
- -0.5
- 0.5
- 4
- 4
- 0
- 12

14. **(1 pt) Library/Rochester/setLinearAlgebra22SymmetricMatrices-/ur_la_22_6.pg**

The matrix $M = \begin{bmatrix} -3 & 0 & 1 & 0 \\ 0 & -3 & 0 & 1 \\ 1 & 0 & -3 & 0 \\ 0 & 1 & 0 & -3 \end{bmatrix}$.

has two distinct eigenvalues $\lambda_1 < \lambda_2$. Find the eigenvalues and an orthonormal basis for each eigenspace.

λ_1 = ____,

associated unit eigenvector = $\begin{bmatrix} \_ \\ \_ \\ \_ \\ \_ \end{bmatrix}$, $\begin{bmatrix} \_ \\ \_ \\ \_ \\ \_ \end{bmatrix}$,

λ_2 = ____,

associated unit eigenvector = $\begin{bmatrix} \_ \\ \_ \\ \_ \\ \_ \end{bmatrix}$, $\begin{bmatrix} \_ \\ \_ \\ \_ \\ \_ \end{bmatrix}$.

The above eigenvectors form an orthonormal eigenbasis for M.

Correct Answers:

- -4
- ```
  \(\displaystyle\left.\begin{array}{c}
  \mbox{0.5} \cr
  \mbox{-0.5} \cr
  \mbox{-0.5} \cr
  \mbox{0.5} \cr
  \end{array}\right.\) ,\(\displaystyle\left.\begin{array}{c
  \mbox{0.5} \cr
  \mbox{0.5} \cr
  \mbox{-0.5} \cr
  \mbox{-0.5} \cr
  \end{array}\right.\)
  ```
- -2
- ```
  \(\displaystyle\left.\begin{array}{c}
  \mbox{-0.5} \cr
  \mbox{0.5} \cr
  \mbox{-0.5} \cr
  \mbox{0.5} \cr
  \end{array}\right.\) ,\(\displaystyle\left.\begin{array}{c
  \mbox{0.5} \cr
  \mbox{0.5} \cr
  \mbox{0.5} \cr
  \mbox{0.5} \cr
  \end{array}\right.\)
  ```

15. **(1 pt) Library/Rochester/setLinearAlgebra11Eigenvalues-/ur_la_11_2.pg**

Find the characteristic polynomial $p(x)$ of the matrix

$$A = \begin{bmatrix} -3 & 2 & 0 \\ 0 & 3 & 1 \\ -1 & 5 & 0 \end{bmatrix}$$

$p(x) =$ ____________

Correct Answers:

- 14*x-x^3+13

16. **(1 pt) Library/Rochester/setLinearAlgebra11Eigenvalues-/ur_la_11_15.pg**

The matrix $A = \begin{bmatrix} 8 & k \\ -4 & -4 \end{bmatrix}$

has two distinct real eigenvalues if and only if $k <$ ____.

Correct Answers:

- 9

17. **(1 pt) Library/TCNJ/TCNJ_Eigenvalues/problem13.pg**

Find the eigenvalues $\lambda_1 < \lambda_2 < \lambda_3$ and corresponding eigenvectors of the matrix

$$A = \begin{bmatrix} -5 & 24 & 24 \\ 0 & 1 & -3 \\ 0 & 0 & 4 \end{bmatrix}.$$

The eigenvalue $\lambda_1 =$___ corresponds to the eigenvector $\begin{bmatrix} \_\_\_ \\ \_\_\_ \\ \_\_\_ \end{bmatrix}$.

The eigenvalue $\lambda_2 =$___ corresponds to the eigenvector $\begin{bmatrix} \_\_\_ \\ \_\_\_ \\ \_\_\_ \end{bmatrix}$.

The eigenvalue $\lambda_3 =$___ corresponds to the eigenvector $\begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}$.

Correct Answers:

- -5
- $\begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}$
- 1
- $\begin{bmatrix} 4 \\ 1 \\ 0 \end{bmatrix}$
- 4
- $\begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$

---

18. **(1 pt) UI/ur_la_11_20a.pg**

The matrix $A = \begin{bmatrix} 2 & 0 & -2 \\ 0 & 4 & 0 \\ 2 & 0 & 6 \end{bmatrix}$ has one real eigenvalue. Find this eigenvalue, its multiplicity, and the dimension of the corresponding eigenspace.

eigenvalue = ____,
algebraic multiplicity = ____,
dimension of the eigenspace = ____.
Is the matrix A defective? (Type "yes" or "no") ____.

Correct Answers:

- 4
- 3
- 2
- YES

---

19. **(1 pt) Library/TCNJ/TCNJ_Diagonalization/problem4.pg**

Let: $A = \begin{bmatrix} -1 & 6 \\ -9 & 14 \end{bmatrix}$

Find S, D and S^{-1} such that $A = SDS^{-1}$.

$S = \begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix}$, $D = \begin{bmatrix} \_\_ & 0 \\ 0 & \_\_ \end{bmatrix}$, $S^{-1} = \begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix}$
.

Correct Answers:

- 1; -2; 1; -3; 5; 8; 1; -2; 1; -3

---

20. **(1 pt) Library/TCNJ/TCNJ_Diagonalization/problem5.pg**

Let $A = \begin{bmatrix} 6 & -3 & 12 \\ -6 & 3 & -12 \\ -3 & 3 & -9 \end{bmatrix}$.

Find S and D such that $A = SDS^{-1}$. $S = \begin{bmatrix} \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \end{bmatrix}$,

$D = \begin{bmatrix} \_\_ & 0 & 0 \\ 0 & \_\_ & 0 \\ 0 & 0 & \_\_ \end{bmatrix}$.

Correct Answers:

- 1; 1; 2; -1; -2; -2; -1; -1; -1; -3; 0; 3

---

21. **(1 pt) Library/Rochester/setLinearAlgebra11Eigenvalues-/ur_la_11_11.pg**

Find a 2×2 matrix A such that
$\begin{bmatrix} -1 \\ -2 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ 0 \end{bmatrix}$
are eigenvectors of A, with eigenvalues 8 and -3 respectively.

$A = \begin{bmatrix} \_\_\_\_ & \_\_\_\_ \\ \_\_\_\_ & \_\_\_\_ \end{bmatrix}$.

Correct Answers:

- -3
- 5.5
- 0
- 8

---

22. **(1 pt) local/Library/UI/LinearSystems/diag.pg**

Given that the matrix A has eigenvalue $\lambda_1 = 8$ with corresponding eigenvector $\begin{bmatrix} 2 \\ -5 \end{bmatrix}$

and eigenvalue $\lambda_2 = -1$ with corresponding eigenvector $\begin{bmatrix} -4 \\ 5 \end{bmatrix}$, find A.

$A = \begin{bmatrix} \_\_\_\_ & \_\_\_\_ \\ \_\_\_\_ & \_\_\_\_ \end{bmatrix}$.

Correct Answers:

- -10
- -7.2
- 22.5
- 17