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1. **(1 pt) Library/Rochester/setLinearAlgebra11Eigenvalues-/ur_la_11_18.pg**

The matrix $A = \begin{bmatrix} 3 & 0 & 2 \\ 0 & 1 & 0 \\ -2 & 0 & -1 \end{bmatrix}$

has one real eigenvalue. Find this eigenvalue and a basis of the eigenspace.

eigenvalue = ____,

Basis: $\begin{bmatrix} \_ \\ \_ \\ \_ \end{bmatrix}$, $\begin{bmatrix} \_ \\ \_ \\ \_ \end{bmatrix}$.

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2. **(1 pt) Library/Rochester/setLinearAlgebra12Diagonalization-/ur_la_12_2.pg**

Let $M = \begin{bmatrix} 8 & -4 \\ 8 & -4 \end{bmatrix}$.

Find formulas for the entries of M^n, where n is a positive integer.

$M^n = \begin{bmatrix} \_\_\_\_ & \_\_\_\_ \\ \_\_\_\_ & \_\_\_\_ \end{bmatrix}$.

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3. **(1 pt) Library/Rochester/setLinearAlgebra14TransfOfRn-/ur_la_14_18.pg**

Let L be the line in $\mathbb{R}^3$ that consists of all scalar multiples of the vector $\begin{bmatrix} -2 \\ 2 \\ 1 \end{bmatrix}$. Find the orthogonal projection of the vector $v = \begin{bmatrix} 9 \\ 3 \\ 8 \end{bmatrix}$ onto L.

$\text{proj}_L v = \begin{bmatrix} \_\_\_ \\ \_\_\_ \\ \_\_\_ \end{bmatrix}$.

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4. **(1 pt) Library/Rochester/setLinearAlgebra17DotProductRn-/ur_la_17_6.pg**

Find a vector v perpendicular to the vector $u = \begin{bmatrix} -4 \\ 2 \end{bmatrix}$.

$v = \begin{bmatrix} \_ \\ \_ \end{bmatrix}$.

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5. **(1 pt) Library/Rochester/setLinearAlgebra17DotProductRn-/ur_la_17_7.pg**

Find the value of k for which the vectors

$x = \begin{bmatrix} -4 \\ -4 \\ -5 \\ 4 \end{bmatrix}$ and $y = \begin{bmatrix} 4 \\ 5 \\ -2 \\ k \end{bmatrix}$ are orthogonal.

$k =$ ____.

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6. **(1 pt) Library/Rochester/setLinearAlgebra17DotProductRn-/ur_la_17_21.pg**

Let $v_1 = \begin{bmatrix} 0.5 \\ 0.5 \\ -0.5 \\ 0.5 \end{bmatrix}$, $v_2 = \begin{bmatrix} 0.5 \\ 0.5 \\ 0.5 \\ -0.5 \end{bmatrix}$, and $v_3 = \begin{bmatrix} -0.5 \\ 0.5 \\ 0.5 \\ 0.5 \end{bmatrix}$.

Find a vector v_4 in $\mathbb{R}^4$ such that the vectors v_1, v_2, v_3, and v_4 are orthonormal.

$v_4 = \begin{bmatrix} \_ \\ \_ \\ \_ \\ \_ \end{bmatrix}$.

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7. **(1 pt) Library/Rochester/setLinearAlgebra18OrthogonalBases-/ur_la_18_4.pg**

Let $x = \begin{bmatrix} -2 \\ -3 \\ 1 \\ 0 \end{bmatrix}$ and $y = \begin{bmatrix} 0 \\ -3 \\ 5 \\ 4 \end{bmatrix}$.

Use the Gram-Schmidt process to determine an orthonormal basis for the subspace of $\mathbb{R}^4$ spanned by x and y.

$\begin{bmatrix} \_\_\_ \\ \_\_\_ \\ \_\_\_ \\ \_\_\_ \end{bmatrix}$, $\begin{bmatrix} \_\_\_ \\ \_\_\_ \\ \_\_\_ \\ \_\_\_ \end{bmatrix}$.

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8. **(1 pt) Library/Rochester/setLinearAlgebra22SymmetricMatrices-/ur_la_22_3.pg**

Find the eigenvalues and associated unit eigenvectors of the (symmetric) matrix

$A = \begin{bmatrix} 4 & -12 \\ -12 & 36 \end{bmatrix}$.

smaller eigenvalue = ____,

associated unit eigenvector = $\begin{bmatrix} \_ \\ \_ \end{bmatrix}$,

larger eigenvalue = ____,

associated unit eigenvector = $\begin{bmatrix} \_ \\ \_ \end{bmatrix}$.

The above eigenvectors form an orthonormal eigenbasis for A.

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9. **(1 pt) Library/TCNJ/TCNJ_OrthogonalSets/problem9.pg**

Given $v = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$, find the coordinates for v in the subspace W spanned by $u_1 = \begin{bmatrix} 7 \\ -1 \end{bmatrix}$ and $u_2 = \begin{bmatrix} 6 \\ 42 \end{bmatrix}$. Note that u_1 and u_2 are orthogonal.

$v =$ ____ $u_1 +$ ____ u_2

10. **(1 pt) Library/Rochester/setLinearAlgebra18OrthogonalBases-/ur_la_18_7.pg**

Let $A = \begin{bmatrix} -2 & 9 & 6 & 6 \\ 3 & 6 & -9 & 4 \end{bmatrix}$.

Find an orthonormal basis of the kernel of A.

$\begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}, \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}$.

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11. **(1 pt) Library/Rochester/setLinearAlgebra18OrthogonalBases-/ur_la_18_11.pg**

Let $A = \begin{bmatrix} 1 & 0 & -4 \\ -2 & -4 & 4 \\ 2 & 5 & -3 \end{bmatrix}$.

Find an orthonormal basis of the column space of A.

$\begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}, \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}$.

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12. **(1 pt) Library/Rochester/setLinearAlgebra19QRfactorization-/ur_la_19_3.pg**

Find the QR factorization of $M = \begin{bmatrix} 4 & -4 & 4 \\ 2 & -5 & 11 \\ 4 & 2 & 4 \end{bmatrix}$.

$M = \begin{bmatrix} \_\_\_ & \_\_\_ & \_\_\_ \\ \_\_\_ & \_\_\_ & \_\_\_ \\ \_\_\_ & \_\_\_ & \_\_\_ \end{bmatrix} \begin{bmatrix} \_\_\_ & \_\_\_ & \_\_\_ \\ \_\_\_ & \_\_\_ & \_\_\_ \\ \_\_\_ & \_\_\_ & \_\_\_ \end{bmatrix}$.

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13. **(1 pt) Library/Rochester/setLinearAlgebra19QRfactorization-/ur_la_19_4.pg**

Find the QR factorization of $M = \begin{bmatrix} 2 & 8 \\ -2 & 4 \\ -2 & -8 \\ -2 & 4 \end{bmatrix}$.

$M = \begin{bmatrix} \_\_\_ & \_\_\_ \\ \_\_\_ & \_\_\_ \\ \_\_\_ & \_\_\_ \\ \_\_\_ & \_\_\_ \end{bmatrix} \begin{bmatrix} \_\_\_ & \_\_\_ \\ \_\_\_ & \_\_\_ \end{bmatrix}$.

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14. **(1 pt) Library/Rochester/setLinearAlgebra22SymmetricMatrices-/ur_la_22_6.pg**

The matrix $M = \begin{bmatrix} -3 & 0 & 1 & 0 \\ 0 & -3 & 0 & 1 \\ 1 & 0 & -3 & 0 \\ 0 & 1 & 0 & -3 \end{bmatrix}$.

has two distinct eigenvalues $\lambda_1 < \lambda_2$. Find the eigenvalues and an orthonormal basis for each eigenspace.

λ_1 = ___,

associated unit eigenvector = $\begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}, \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}$,

λ_2 = ___,

associated unit eigenvector = $\begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}, \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}$.

The above eigenvectors form an orthonormal eigenbasis for M.

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15. **(1 pt) Library/Rochester/setLinearAlgebra11Eigenvalues-/ur_la_11_2.pg**

Find the characteristic polynomial $p(x)$ of the matrix

$$A = \begin{bmatrix} -3 & 2 & 0 \\ 0 & 3 & 1 \\ -1 & 5 & 0 \end{bmatrix}$$

$p(x) =$ ______________

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16. **(1 pt) Library/Rochester/setLinearAlgebra11Eigenvalues-/ur_la_11_15.pg**

The matrix $A = \begin{bmatrix} 8 & k \\ -4 & -4 \end{bmatrix}$

has two distinct real eigenvalues if and only if $k <$ ___.

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17. **(1 pt) Library/TCNJ/TCNJ_Eigenvalues/problem13.pg**

Find the eigenvalues $\lambda_1 < \lambda_2 < \lambda_3$ and corresponding eigenvectors of the matrix

$$A = \begin{bmatrix} -5 & 24 & 24 \\ 0 & 1 & -3 \\ 0 & 0 & 4 \end{bmatrix}.$$

The eigenvalue $\lambda_1 =$ ___ corresponds to the eigenvector $\begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}$.

The eigenvalue $\lambda_2 =$ ___ corresponds to the eigenvector $\begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}$.

The eigenvalue $\lambda_3 =$ ___ corresponds to the eigenvector $\begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}$.

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18. **(1 pt) UI/ur_la_11_20a.pg**

The matrix $A = \begin{bmatrix} 2 & 0 & -2 \\ 0 & 4 & 0 \\ 2 & 0 & 6 \end{bmatrix}$

has one real eigenvalue. Find this eigenvalue, its multiplicity, and the dimension of the corresponding eigenspace.

eigenvalue = ___,

algebraic multiplicity = ___,

dimension of the eigenspace = ___.

Is the matrix A defective? (Type "yes" or "no") ___.

19. **(1 pt) Library/TCNJ/TCNJ_Diagonalization/problem4.pg**

Let: $A = \begin{bmatrix} -1 & 6 \\ -9 & 14 \end{bmatrix}$

Find S, D and S^{-1} such that $A = SDS^{-1}$.

$S = \begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix}$, $D = \begin{bmatrix} \_\_ & 0 \\ 0 & \_\_ \end{bmatrix}$, $S^{-1} = \begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix}$
.

20. **(1 pt) Library/TCNJ/TCNJ_Diagonalization/problem5.pg**

Let $A = \begin{bmatrix} 6 & -3 & 12 \\ -6 & 3 & -12 \\ -3 & 3 & -9 \end{bmatrix}$.

Find S and D such that $A = SDS^{-1}$. $S = \begin{bmatrix} \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \end{bmatrix}$,

$D = \begin{bmatrix} \_\_ & 0 & 0 \\ 0 & \_\_ & 0 \\ 0 & 0 & \_\_ \end{bmatrix}$.

21. **(1 pt) Library/Rochester/setLinearAlgebra11Eigenvalues-/ur_la_11_11.pg**

Find a 2×2 matrix A such that

$\begin{bmatrix} -1 \\ -2 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ 0 \end{bmatrix}$

are eigenvectors of A, with eigenvalues 8 and -3 respectively.

$A = \begin{bmatrix} \_\_\_\_ & \_\_\_\_ \\ \_\_\_\_ & \_\_\_\_ \end{bmatrix}$.

22. **(1 pt) local/Library/UI/LinearSystems/diag.pg**

Given that the matrix A has eigenvalue $\lambda_1 = 8$ with corresponding eigenvector $\begin{bmatrix} 2 \\ -5 \end{bmatrix}$

and eigenvalue $\lambda_2 = -1$ with corresponding eigenvector $\begin{bmatrix} -4 \\ 5 \end{bmatrix}$, find A.

$A = \begin{bmatrix} \_\_\_\_ & \_\_\_\_ \\ \_\_\_\_ & \_\_\_\_ \end{bmatrix}$.