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**1.** **(1 pt) local/Library/UI/eigenTF.pg**
$A$ is $n \times n$ an matrices.

Check the true statements below:

- A. There are an infinite number of eigenvectors that correspond to a particular eigenvalue of $A$.
- B. The vector $\mathbf{0}$ is an eigenvector of $A$ if and only if the columns of $A$ are linearly dependent.
- C. The vector $\mathbf{0}$ can never be an eigenvector of $A$
- D. 0 can never be an eigenvalue of $A$.
- E. $A$ will have at most $n$ eigenvectors.
- F. The vector $\mathbf{0}$ is an eigenvector of $A$ if and only if $Ax = 0$ has a nonzero solution
- G. 0 is an eigenvalue of $A$ if and only if $Ax = 0$ has a nonzero solution
- H. $A$ will have at most $n$ eigenvalues.
- I. 0 is an eigenvalue of $A$ if and only if det($A$) = 0
- J. 0 is an eigenvalue of $A$ if and only if $Ax = 0$ has an infinite number of solutions
- K. 0 is an eigenvalue of $A$ if and only if the columns of $A$ are linearly dependent.
- L. The eigenspace corresponding to a particular eigenvalue of $A$ contains an infinite number of vectors.
- M. The vector $\mathbf{0}$ is an eigenvector of $A$ if and only if det($A$) = 0

*Correct Answers:*

- `ACGHIJKL`

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**2.** **(1 pt) UI/DIAGtfproblem1.pg**
$A$, $P$ and $D$ are $n \times n$ matrices.

Check the true statements below:

- A. $A$ is diagonalizable if $A$ has $n$ distinct linearly independent eigenvectors.
- B. $A$ is diagonalizable if and only if $A$ has $n$ eigenvalues, counting multiplicities.
- C. If there exists a basis for $\mathbb{R}^n$ consisting entirely of eigenvectors of $A$, then $A$ is diagonalizable.
- D. If $A$ is diagonalizable, then $A$ is symmetric.
- E. $A$ is diagonalizable if $A$ has $n$ distinct eigenvectors.
- F. If $A$ is invertible, then $A$ is diagonalizable.
- G. $A$ is diagonalizable if $A = PDP^{-1}$ for some diagonal matrix $D$ and some invertible matrix $P$.
- H. If $A$ is orthogonally diagonalizable, then $A$ is symmetric.
- I. If $A$ is diagonalizable, then $A$ is invertible.
- J. If $AP = PD$, with $D$ diagonal, then the nonzero columns of $P$ must be eigenvectors of $A$.
- K. If $A$ is diagonalizable, then $A$ has $n$ distinct eigenvalues.
- L. If $A$ is symmetric, then $A$ is orthogonally diagonalizable.
- M. If $A$ is symmetric, then $A$ is diagonalizable.

*Correct Answers:*

- `ACGHJLM`

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**3.** **(1 pt) UI/orthog.pg**
All vectors and subspaces are in $\mathbb{R}^n$.

Check the true statements below:

- A. If $\{v_1, v_2, v_3\}$ is an orthonormal set, then the set $\{v_1, v_2, v_3\}$ is linearly independent.
- B. If $x$ is not in a subspace $W$, then $x - \text{proj}_W(x)$ is not zero.
- C. If $W = Span\{x_1, x_2, x_3\}$ and if $\{v_1, v_2, v_3\}$ is an orthonormal set in $W$, then $\{v_1, v_2, v_3\}$ is an orthonormal basis for $W$.
- D. If $A$ is symmetric, $A\mathbf{v} = r\mathbf{v}$, $A\mathbf{w} = s\mathbf{w}$ and $r \neq s$, then $\mathbf{v} \cdot \mathbf{w} = 0$ .
- E. If $\mathbf{v}$ and $\mathbf{w}$ are both eigenvectors of $A$ and if $A$ is symmetric, then $\mathbf{v} \cdot \mathbf{w} = 0$ .
- F. If $A\mathbf{v} = r\mathbf{v}$ and $A\mathbf{w} = s\mathbf{w}$ and $r \neq s$, then $\mathbf{v} \cdot \mathbf{w} = 0$ .
- G. In a $QR$ factorization, say $A = QR$ (when $A$ has linearly independent columns), the columns of $Q$ form an orthonormal basis for the column space of $A$.

*Correct Answers:*

- `ABCDG`

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Suppose $A = PDP^{-1}$ where $D$ is a diagonal matrix. If $P = [\ \vec{p}_1\ \vec{p}_2\ \vec{p}_3\ ]$, then $2\vec{p}_1$ is an eigenvector of $A$

- A. True
- B. False

*Correct Answers:*

- `A`

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Suppose $A = PDP^{-1}$ where $D$ is a diagonal matrix. If $P = [\ \vec{p}_1\ \vec{p}_2\ \vec{p}_3\ ]$, then $\vec{p}_1 + \vec{p}_2$ is an eigenvector of $A$

- A. True
- B. False

*Correct Answers:*

- `B`

Suppose $A = PDP^{-1}$ where $D$ is a diagonal matrix. Suppose also the $d_{ii}$ are the diagonal entries of D. If $P = [\ \vec{p}_1\ \vec{p}_2\ \vec{p}_3\ ]$ and $d_{11} = d_{22}$, then $\vec{p}_1 + \vec{p}_2$ is an eigenvector of $A$

- A. True
- B. False

*Correct Answers:*

- A

---

Suppose $A = PDP^{-1}$ where $D$ is a diagonal matrix. Suppose also the $d_{ii}$ are the diagonal entries of D. If $P = [\ \vec{p}_1\ \vec{p}_2\ \vec{p}_3\ ]$ and $d_{22} = d_{33}$, then $\vec{p}_1 + \vec{p}_2$ is an eigenvector of $A$

- A. True
- B. False

*Correct Answers:*

- B

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If $\vec{v}_1$ and $\vec{v}_2$ are eigenvectors of $A$ corresponding to eigenvalue $\lambda_0$, then $6\vec{v}_1 - 8\vec{v}_2$ is also an eigenvector of $A$ corresponding to eigenvalue $\lambda_0$ when $6\vec{v}_1 - 8\vec{v}_2$ is not $\vec{0}$ .

- A. True
- B. False

**Hint:** *(Instructor hint preview: show the student hint after 0 attempts. The current number of attempts is 0. )*
Is an eigenspace a subspace? Is an eigenspace closed under linear combinations?
Also, is $6\vec{v}_1 - 8\vec{v}_2$ nonzero?

*Correct Answers:*

- A

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Which of the following is an eigenvalue of $\begin{bmatrix} 4 & 4 \\ 1 & 4 \end{bmatrix}$.

- A. -4
- B. -3
- C. -2
- D. -1
- E. 0
- F. 1
- G. 2
- H. 3
- I. 4
- J. none of the above

*Correct Answers:*

- G

---

Let $A = \begin{bmatrix} 6 & 1 & -1 \\ 0 & -6 & -8 \\ 0 & 0 & 6 \end{bmatrix}$. Is $A$ = diagonalizable?

- A. yes
- B. no
- C. none of the above

**Solution:** *(Instructor solution preview: show the student solution after due date. )*

SOLUTION
Note that since the matrix is triangular, the eigenvalues are the diagonal elements 6 and -6. Since $A$ is a 3 x 3 matrix, we need 3 linearly independent eigenvectors. Since -6 has algebraic multiplicity 1, it has geometric multiplicity 1 (the dimension of its eigenspace is 1). Thus we can only use one eigenvector from the eigenspace corresponding to eigenvalue -6 to form $P$.

Thus we need 2 linearly independent eigenvectors from the eigenspace corresponding to the other eigenvalue 6. The eigenvalue 6 has algebraic multiplicity 2. Let E = dimension of the eigenspace corresponding eigenvalue 6. Then $1 \leq E \leq 2$. But we can easily see that the Nullspace of $A - 6I$ has dimension 1.

Thus we do not have enough linearly independent eigenvectors to form $P$. Hence $A$ is not diagonalizable.

*Correct Answers:*

- B

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Let $A = \begin{bmatrix} 5 & -44 & -12 \\ 0 & -6 & -3 \\ 0 & 0 & 5 \end{bmatrix}$. Is $A$ = diagonalizable?

- A. yes
- B. no
- C. none of the above

**Solution:** *(Instructor solution preview: show the student solution after due date. )*

SOLUTION
Note that since the matrix is triangular, the eigenvalues are the diagonal elements 5 and -6. Since $A$ is a 3 x 3 matrix, we need 3 linearly independent eigenvectors. Since -6 has algebraic multiplicity 1, it has geometric multiplicity 1 (the dimension of its eigenspace is 1). Thus we can only use one eigenvector from the eigenspace corresponding to eigenvalue -6 to form $P$.

Thus we need 2 linearly independent eigenvectors from the eigenspace corresponding to the other eigenvalue 5. The eigenvalue 5 has algebraic multiplicity 2. Let E = dimension of the eigenspace corresponding eigenvalue 5. Then $1 \leq E \leq 2$. But we can easily see that the Nullspace of $A-5I$ has dimension 2.

Thus we have 3 linearly independent eigenvectors which we can use to form the square matrix $P$. Hence $A$ is diagonalizable.

*Correct Answers:*

- A

---

Let $A = \begin{bmatrix} 7 & 11 \\ 4 & -4 \end{bmatrix}$. Is $A$ = diagonalizable?

- A. yes
- B. no
- C. none of the above

**Hint:** *(Instructor hint preview: show the student hint after 0 attempts. The current number of attempts is 0. )*

You do NOT need to do much work for this problem. You just need to know if the matrix $A$ is diagonalizable. Since $A$ is a 2 x 2 matrix, you need 2 linearly independent eigenvectors of $A$ to form $P$. Does $A$ have 2 linearly independent eigenvectors? Note you don't need to know what these eigenvectors are. You don't even need to know the eigenvalues.

**Solution:** *(Instructor solution preview: show the student solution after due date. )*

SOLUTION

The matrix $A$ has 2 eigenvalues (note you do not need to know their values, just that you have 2 distinct eigenvalues). Hence the $A$ is diagonalizable by the following: Each eigenvalue has a 1-dimensional eigenspace. If one takes one eigenvector (any nonzero element of the eigenspace) from each eigenspace, then that pair forms a linearly independent set and can be used to form $P$. Since $A$ is a 2x2 matrix, one only needs 2 linearly independent eigenvectors to form $P$.

Sidenote 1: This works whenever one has n distinct eigenvalues for an n x n matrix. The only time a matrix is not diagonalizable is when there exists an eigenvalue whose geometric multiplicity is strictly less than its algebraic multiplicity. Then you will not have enough linearly independent eigenvectors to form $P$.

Sidenote 2: This works even if the eigenvalues are complex number and not real numbers, but we won't handle that case in this class (but the algorithm is identical to that for real eigenvalues).

*Correct Answers:*

- A

---

Let $A = \begin{bmatrix} 0.611111111111111 & -6.77777777777778 & -4.8888888 \\ -1.27777777777778 & 1.44444444444444 & 0.277777777 \\ -3.05555555555556 & -6.11111111111111 & 4.33333333 \end{bmatrix}$

and let $P = \begin{bmatrix} 2 & 9 & -4 \\ -1 & 2 & 1 \\ 0 & 5 & 2 \end{bmatrix}$.

Suppose $A = PDP^{-1}$. Then if $d_{ii}$ are the diagonal entries of $D$, $d_{11} =$,

- A. -4
- B. -3
- C. -2
- D. -1
- E. 0
- F. 1
- G. 2
- H. 3
- I. 4
- J. 5

**Hint:** *(Instructor hint preview: show the student hint after 0 attempts. The current number of attempts is 0. )*

Use definition of eigenvalue since you know an eigenvector corresponding to eigenvalue $d_{11}$.

*Correct Answers:*

- I

---

Calculate the dot product: $\begin{bmatrix} 4 \\ -2 \\ -4 \end{bmatrix} \cdot \begin{bmatrix} -4 \\ 2 \\ -5 \end{bmatrix}$

- A. -4
- B. -3
- C. -2
- D. -1
- E. 0
- F. 1
- G. 2
- H. 3
- I. 4
- J. none of the above

*Correct Answers:*

- E

---

Suppose $A \begin{bmatrix} 2 \\ 1 \\ -3 \end{bmatrix} = \begin{bmatrix} -4 \\ -2 \\ 6 \end{bmatrix}$. Then an eigenvalue of $A$ is

- A. -4
- B. -3
- C. -2

- D. -1
- E. 0
- F. 1
- G. 2
- H. 3
- I. 4
- J. none of the above

*Correct Answers:*

- `C`

---

Determine the length of $\begin{bmatrix} 1 \\ 3.87298334620742 \end{bmatrix}$.

- A. -4
- B. -3
- C. -2
- D. -1
- E. 0
- F. 1
- G. 2
- H. 3
- I. 4
- J. 5

*Correct Answers:*

- `I`

---

If the characteristic polynomial of $A = (\lambda-8)^9(\lambda-3)(\lambda-5)^5$, then the algebraic multiplicity of $\lambda = 3$ is

- A. -4
- B. -3
- C. -2
- D. -1
- E. 0
- F. 1
- G. 2
- H. 3
- I. 4
- J. none of the above

*Correct Answers:*

- `F`

---

If the characteristic polynomial of $A = (\lambda+2)^4(\lambda-7)(\lambda+6)^3$, then the geometric multiplicity of $\lambda = 7$ is

- A. -4
- B. -3
- C. -2
- D. -1
- E. 0
- F. 1
- G. 2
- H. 3
- I. 4
- J. none of the above

*Correct Answers:*

- `F`

---

If the characteristic polynomial of $A = (\lambda-6)^5(\lambda-5)^2(\lambda-3)^5$, then the algebraic multiplicity of $\lambda = 5$ is

- A. 0
- B. 1
- C. 2
- D. 3
- E. 0 or 1
- F. 0 or 2
- G. 1 or 2
- H. 0, 1, or 2
- I. 0, 1, 2, or 3
- J. none of the above

*Correct Answers:*

- `C`

---

If the characteristic polynomial of $A = (\lambda+3)^4(\lambda+6)^2(\lambda-3)^4$, then the geometric multiplicity of $\lambda = -6$ is

- A. 0
- B. 1
- C. 2
- D. 3
- E. 0 or 1
- F. 0 or 2
- G. 1 or 2
- H. 0, 1, or 2
- I. 0, 1, 2, or 3
- J. none of the above

*Correct Answers:*

- `G`

---

Suppose the orthogonal projection of $\begin{bmatrix} -35 \\ 5 \\ 6 \end{bmatrix}$ onto $\begin{bmatrix} 1 \\ 1 \\ -4 \end{bmatrix}$ is $(z_1, z_2, z_3)$. Then $z_1 =$

- A. -4
- B. -3
- C. -2
- D. -1
- E. 0
- F. 1
- G. 2
- H. 3

- I. 4
- J. none of the above

*Correct Answers:*

- B

---

Suppose $\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$ is a unit vector in the direction of $\begin{bmatrix} -1 \\ -5 \\ 4.81894409826699 \end{bmatrix}$. Then $u_1 =$

- A. -0.8
- B. -0.6
- C. -0.4
- D. -0.2
- E. 0
- F. 0.2
- G. 0.4
- H. 0.6
- I. 0.8
- J. 1

*Correct Answers:*

- B