1. **(1 pt) local/Library/UI/eigenTF.pg**
A is $n \times n$ an matrices.

Check the true statements below:

- A. There are an infinite number of eigenvectors that correspond to a particular eigenvalue of A.
- B. The vector $\mathbf{0}$ is an eigenvector of A if and only if the columns of A are linearly dependent.
- C. The vector $\mathbf{0}$ can never be an eigenvector of A
- D. 0 can never be an eigenvalue of A.
- E. A will have at most n eigenvectors.
- F. The vector $\mathbf{0}$ is an eigenvector of A if and only if $Ax = 0$ has a nonzero solution
- G. 0 is an eigenvalue of A if and only if $Ax = 0$ has a nonzero solution
- H. A will have at most n eigenvalues.
- I. 0 is an eigenvalue of A if and only if det(A) = 0
- J. 0 is an eigenvalue of A if and only if $Ax = 0$ has an infinite number of solutions
- K. 0 is an eigenvalue of A if and only if the columns of A are linearly dependent.
- L. The eigenspace corresponding to a particular eigenvalue of A contains an infinite number of vectors.
- M. The vector $\mathbf{0}$ is an eigenvector of A if and only if det(A) = 0

2. **(1 pt) UI/DIAGtfproblem1.pg**
A, P and D are $n \times n$ matrices.

Check the true statements below:

- A. A is diagonalizable if A has n distinct linearly independent eigenvectors.
- B. A is diagonalizable if and only if A has n eigenvalues, counting multiplicities.
- C. If there exists a basis for $\mathbb{R}^n$ consisting entirely of eigenvectors of A, then A is diagonalizable.
- D. If A is diagonalizable, then A is symmetric.
- E. A is diagonalizable if A has n distinct eigenvectors.
- F. If A is invertible, then A is diagonalizable.
- G. A is diagonalizable if $A = PDP^{-1}$ for some diagonal matrix D and some invertible matrix P.
- H. If A is orthogonally diagonalizable, then A is symmetric.
- I. If A is diagonalizable, then A is invertible.
- J. If $AP = PD$, with D diagonal, then the nonzero columns of P must be eigenvectors of A.
- K. If A is diagonalizable, then A has n distinct eigenvalues.
- L. If A is symmetric, then A is orthogonally diagonalizable.
- M. If A is symmetric, then A is diagonalizable.

3. **(1 pt) UI/orthog.pg**
All vectors and subspaces are in $\mathbb{R}^n$.

Check the true statements below:

- A. If $\{v_1, v_2, v_3\}$ is an orthonormal set, then the set $\{v_1, v_2, v_3\}$ is linearly independent.
- B. If x is not in a subspace W, then $x - \text{proj}_W(x)$ is not zero.
- C. If $W = Span\{x_1, x_2, x_3\}$ and if $\{v_1, v_2, v_3\}$ is an orthonormal set in W, then $\{v_1, v_2, v_3\}$ is an orthonormal basis for W.
- D. If A is symmetric, $A\mathbf{v} = r\mathbf{v}$, $A\mathbf{w} = s\mathbf{w}$ and $r \neq s$, then $\mathbf{v} \cdot \mathbf{w} = 0$.
- E. If $\mathbf{v}$ and $\mathbf{w}$ are both eigenvectors of A and if A is symmetric, then $\mathbf{v} \cdot \mathbf{w} = 0$.
- F. If $A\mathbf{v} = r\mathbf{v}$ and $A\mathbf{w} = s\mathbf{w}$ and $r \neq s$, then $\mathbf{v} \cdot \mathbf{w} = 0$.
- G. In a QR factorization, say $A = QR$ (when A has linearly independent columns), the columns of Q form an orthonormal basis for the column space of A.

Suppose $A = PDP^{-1}$ where D is a diagonal matrix. If $P = [\ \vec{p}_1\ \vec{p}_2\ \vec{p}_3\ ]$, then $2\vec{p}_1$ is an eigenvector of A

- A. True
- B. False

Suppose $A = PDP^{-1}$ where D is a diagonal matrix. If $P = [\ \vec{p}_1\ \vec{p}_2\ \vec{p}_3\ ]$, then $\vec{p}_1 + \vec{p}_2$ is an eigenvector of A

- A. True
- B. False

Suppose $A = PDP^{-1}$ where D is a diagonal matrix. Suppose also the d_{ii} are the diagonal entries of D. If $P = [\ \vec{p}_1\ \vec{p}_2\ \vec{p}_3\ ]$ and $d_{11} = d_{22}$, then $\vec{p}_1 + \vec{p}_2$ is an eigenvector of A

- A. True
- B. False

Suppose $A = PDP^{-1}$ where D is a diagonal matrix. Suppose also the d_{ii} are the diagonal entries of D. If $P = [\ \vec{p}_1\ \vec{p}_2\ \vec{p}_3\ ]$ and $d_{22} = d_{33}$, then $\vec{p}_1 + \vec{p}_2$ is an eigenvector of A

- A. True
- B. False

If $\vec{v}_1$ and $\vec{v}_2$ are eigenvectors of A corresponding to eigenvalue λ_0, then $6\vec{v}_1 - 8\vec{v}_2$ is also an eigenvector of A corresponding to eigenvalue λ_0 when $6\vec{v}_1 - 8\vec{v}_2$ is not $\vec{0}$.

- A. True
- B. False

Hint: *(Instructor hint preview: show the student hint after 0 attempts. The current number of attempts is 0.)*
Is an eigenspace a subspace? Is an eigenspace closed under linear combinations?
Also, is $6\vec{v}_1 - 8\vec{v}_2$ nonzero?

Which of the following is an eigenvalue of $\begin{bmatrix} 4 & 4 \\ 1 & 4 \end{bmatrix}$.

- A. -4
- B. -3
- C. -2
- D. -1
- E. 0
- F. 1
- G. 2
- H. 3
- I. 4
- J. none of the above

Let $A = \begin{bmatrix} 6 & 1 & -1 \\ 0 & -6 & -8 \\ 0 & 0 & 6 \end{bmatrix}$. Is A = diagonalizable?

- A. yes
- B. no
- C. none of the above

Let $A = \begin{bmatrix} 5 & -44 & -12 \\ 0 & -6 & -3 \\ 0 & 0 & 5 \end{bmatrix}$. Is A = diagonalizable?

- A. yes
- B. no
- C. none of the above

Let $A = \begin{bmatrix} 7 & 11 \\ 4 & -4 \end{bmatrix}$. Is A = diagonalizable?

- A. yes
- B. no
- C. none of the above

Hint: *(Instructor hint preview: show the student hint after 0 attempts. The current number of attempts is 0.)*
You do NOT need to do much work for this problem. You just need to know if the matrix A is diagonalizable. Since A is a 2 x 2 matrix, you need 2 linearly independent eigenvectors of A to form P. Does A have 2 linearly independent eigenvectors? Note you don't need to know what these eigenvectors are. You don't even need to know the eigenvalues.

Let $A = \begin{bmatrix} 0.611111111111111 & -6.77777777777778 & -4.88888888 \\ -1.27777777777778 & 1.44444444444444 & 0.277777777 \\ -3.05555555555556 & -6.11111111111111 & 4.33333333 \end{bmatrix}$
and let $P = \begin{bmatrix} 2 & 9 & -4 \\ -1 & 2 & 1 \\ 0 & 5 & 2 \end{bmatrix}$.

Suppose $A = PDP^{-1}$. Then if d_{ii} are the diagonal entries of D, $d_{11} =$,

- A. -4
- B. -3
- C. -2
- D. -1
- E. 0
- F. 1
- G. 2
- H. 3
- I. 4
- J. 5

Hint: *(Instructor hint preview: show the student hint after 0 attempts. The current number of attempts is 0.)*
Use definition of eigenvalue since you know an eigenvector corresponding to eigenvalue d_{11}.

Calculate the dot product: $\begin{bmatrix} 4 \\ -2 \\ -4 \end{bmatrix} \cdot \begin{bmatrix} -4 \\ 2 \\ -5 \end{bmatrix}$

- A. -4
- B. -3
- C. -2
- D. -1
- E. 0
- F. 1
- G. 2

- H. 3
- I. 4
- J. none of the above

Suppose $A\begin{bmatrix} 2 \\ 1 \\ -3 \end{bmatrix} = \begin{bmatrix} -4 \\ -2 \\ 6 \end{bmatrix}$. Then an eigenvalue of A is

- A. -4
- B. -3
- C. -2
- D. -1
- E. 0
- F. 1
- G. 2
- H. 3
- I. 4
- J. none of the above

Determine the length of $\begin{bmatrix} 1 \\ 3.87298334620742 \end{bmatrix}$.

- A. -4
- B. -3
- C. -2
- D. -1
- E. 0
- F. 1
- G. 2
- H. 3
- I. 4
- J. 5

If the characteristic polynomial of $A = (\lambda-8)^9(\lambda-3)(\lambda-5)^5$, then the algebraic multiplicity of $\lambda = 3$ is

- A. -4
- B. -3
- C. -2
- D. -1
- E. 0
- F. 1
- G. 2
- H. 3
- I. 4
- J. none of the above

If the characteristic polynomial of $A = (\lambda+2)^4(\lambda-7)(\lambda+6)^3$, then the geometric multiplicity of $\lambda = 7$ is

- A. -4
- B. -3
- C. -2
- D. -1
- E. 0
- F. 1
- G. 2
- H. 3
- I. 4
- J. none of the above

If the characteristic polynomial of $A = (\lambda-6)^5(\lambda-5)^2(\lambda-3)^5$, then the algebraic multiplicity of $\lambda = 5$ is

- A. 0
- B. 1
- C. 2
- D. 3
- E. 0 or 1
- F. 0 or 2
- G. 1 or 2
- H. 0, 1, or 2
- I. 0, 1, 2, or 3
- J. none of the above

If the characteristic polynomial of $A = (\lambda+3)^4(\lambda+6)^2(\lambda-3)^4$, then the geometric multiplicity of $\lambda = -6$ is

- A. 0
- B. 1
- C. 2
- D. 3
- E. 0 or 1
- F. 0 or 2
- G. 1 or 2
- H. 0, 1, or 2
- I. 0, 1, 2, or 3
- J. none of the above

Suppose the orthogonal projection of $\begin{bmatrix} -35 \\ 5 \\ 6 \end{bmatrix}$ onto $\begin{bmatrix} 1 \\ 1 \\ -4 \end{bmatrix}$ is (z_1, z_2, z_3). Then $z_1 =$

- A. -4
- B. -3
- C. -2
- D. -1
- E. 0
- F. 1
- G. 2
- H. 3
- I. 4

- J. none of the above

Suppose $\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$ is a unit vector in the direction of $\begin{bmatrix} -1 \\ -5 \\ 4.81894409826699 \end{bmatrix}$. Then $u_1 =$

- A. -0.8
- B. -0.6
- C. -0.4
- D. -0.2
- E. 0
- F. 0.2
- G. 0.4
- H. 0.6
- I. 0.8
- J. 1