Thm $8'$: If A is a SQUARE $n \times n$ matrix, then the following are equivalent.

a.) A is invertible.

b.) The row-reduced echelon form of A is I_n, the identity matrix.

c.) An echelon form of A has n leading entries [I.e., every column of an echelon form of A is a leading entry column – no free variables]. (A square => A has leading entry in every column if and only if A has leading entry in every row).

d.) The column vectors of A are linearly independent.

e.) $Ax = 0$ has only the trivial solution.

f.) $Ax = b$ has at most one sol'n for any b.

g.) $Ax = b$ has a unique sol'n for any b.

h.) $Ax = b$ is consistent for every $n \times 1$ matrix b.

i.) $Ax = b$ has at least one sol'n for any b.

j.) The column vectors of A span R^n. [every vector in R^n can be written as a linear combination of the columns of A].

k.) There is a square matrix C such that $CA = I$.

l.) There is a square matrix D such that $AD = I$.

m.) A^T is invertible.

n.) A is expressible as a product of elementary matrices.

o.) The column vectors of A form a basis for R^n.
[every vector in R^n can be written uniquely as a linear combination of the columns of A].

p.) Col $A = R^n$.

q.) dim Col $A = n$.

r.) rank of $A = n$.

s.) Nul $A = \{\mathbf{0}\}$,

t.) dim Nul $A = 0$.

u.) A has nullity 0.

Rank(A) + nullity(A) = Number of columns of A.

Ex. 2) Suppose A is a $9X4$ matrix.

If Rank(A) $= 4$, then nullity(A) $=$

$A\mathbf{x} = \mathbf{0}$ has ____________________ solutions.

$A\mathbf{x} = \mathbf{b}$ has ____________________ solutions.

If Rank(A) $= 3$, then nullity(A) $=$

$A\mathbf{x} = \mathbf{0}$ has ____________________ solutions.

$A\mathbf{x} = \mathbf{b}$ has ____________________ solutions.